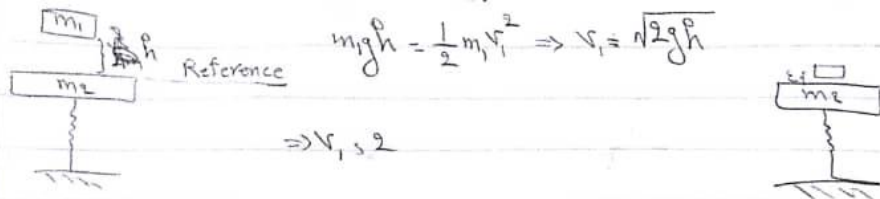


A 30-kg block is dropped from a height of 2 m onto the 10-kg pan of a spring scale. Assuming the impact to be perfectly plastic, determine the maximum deflection of the pan. The constant of the spring is  $k = 20 \text{ kN/m}$ .



$$m_1 g h = \frac{1}{2} m_1 v_1^2 \Rightarrow v_1 = \sqrt{2gh}$$

Step 2: Impact: conservation of momentum

Impact is perfectly plastic  $\Rightarrow e = 0$ ,  $e = \frac{(v_2') - v_1'}{v_1 - v_2}$

$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v \Rightarrow v = \frac{m_1 v_1}{m_1 + m_2} = \frac{m_1}{m_1 + m_2} \sqrt{2gh}$

$\Rightarrow v = \frac{m_1}{m_1 + m_2} \sqrt{2gh} \Rightarrow v = \frac{30}{30 + 10} \sqrt{2 \cdot 9.81 \cdot 2}$

Step 3: conservation of energy ~~before~~ (before and after impact)

The initial deflection of the spring due to weight  $W_2$

$$W_2 = m_2 g \Rightarrow \delta_3 = \frac{W_2}{k} = \frac{m_2 g}{k}$$

$\frac{1}{2} k (\delta_3^2 + (m_1 + m_2) g \delta_{max}) + \frac{1}{2} (m_1 + m_2) v^2 = \frac{1}{2} k (\delta_3 + \delta_{max})^2$

$\Rightarrow \frac{1}{2} k \left( \frac{m_2 g}{k} \right)^2 + (m_1 + m_2) g \delta_{max} + \frac{1}{2} (m_1 + m_2) \left( \frac{m_1}{m_1 + m_2} \sqrt{2gh} \right)^2 = \frac{1}{2} k \left( \frac{m_2 g}{k} + \delta_{max} \right)^2$

$$k \delta_{max}^2 + [2 m_2 g - 2(m_1 + m_2) g] \delta_{max} + \left[ \frac{(m_2 g)^2}{k} - \frac{2 m_1^2 g h}{m_1 + m_2} - \frac{(m_2 g)^2}{k} \right] = 0$$

$$k \delta_{\max}^2 - 2m_1 g \delta_{\max} - \frac{2m_1^2 g h}{m_1 + m_2} = 0 \Rightarrow \delta_{\max} = \frac{-(-2m_1 g) \pm \sqrt{(-2m_1 g)^2 + 8 \frac{m_1^2 g h k}{m_1 + m_2}}}{2k}$$

$$\delta_{\max} = \frac{-(-2m_1 g) \pm \sqrt{(2m_1 g)^2 + 8 \frac{m_1^2 g h k}{m_1 + m_2}}}{2k}$$

$$v_1 = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 2} = 6.2642 \frac{\text{m}}{\text{s}}$$

$$v_2 = \frac{m_1}{m_1 + m_2} \sqrt{2gh} = \frac{30}{30 + 10} \sqrt{2 \times 9.81 \times 2} = 4.6981 \frac{\text{m}}{\text{s}}$$

$$\delta_{\max} = \frac{-(-2 \times 30 \times 9.81) \pm \sqrt{(2 \times 30 \times 9.81)^2 + 8 \frac{30^2 \times 9.81 \times 2 \times 20000}{40}}}{40000} = 0.2253 \text{ m}$$

$$\delta_{\max} = 0.2253 \text{ m}$$