Dynamics midterm q3

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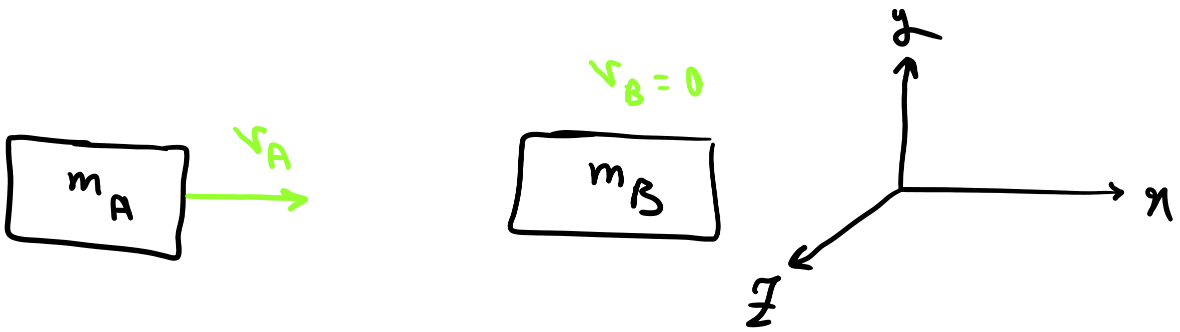
The *mass A* attempts to dock with the mass *B*[redacted]. Their masses are m_A= 18 kg and m_B = 6.6 kg. The mass *B* stationary relative to the reference frame, and the mass *A* approaches velocity V[redacted]$_A$ = (0.2 i + 0.3 j - 0.02 k) m/s.
(a) If the first attempt at docking is successful, what is the velocity of the centre mass of the combined masses afterwards?
(b) If the coefficient of restitution of resulting impact is e = 0.95, what are the velocities of the two masses after impact?



(a)

$$v = \frac{m_A v_A + m_B v_B}{m_A + m_B} = \frac{18(0.2\hat{i} + 0.3\hat{j} - 0.02\hat{k}) + 6.6 \times 0}{18 + 6.6}$$

$$= 0.1463\hat{i} + 0.2195\hat{j} - 0.0146\hat{k} \ \frac{m}{s}$$

$$\boxed{v = 0.1463\hat{i} + 0.2195\hat{j} - 0.0146\hat{k} \ \left(\frac{m}{s}\right)}$$

(b) The y and z component of the velocities of both masses are unchanged.

To determine the x components, we use conservation of linear momentum.

$$m_1 (v_A)_x + m_2 (v_B)_x = m_1 (v'_A)_x + m_2 (v'_B)_x \Rightarrow$$

$$18(0.2) + 6.6 \times (0) = 18 (v'_A)_x + 6.6 (v'_B)_x \Rightarrow$$

$$\boxed{18 (v'_A)_x + 6.6 (v'_B)_x = 3.6} \quad (1)$$

$$e = \frac{(v'_B)_x - (v'_A)_x}{(v_A)_x - (v_B)_x} \Rightarrow 0.95 = \frac{(v'_B)_x - (v'_A)_x}{0.2 - 0} \Rightarrow$$

$$\boxed{-(v'_A)_x + (v'_B)_x = 0.19} \quad (2)$$

$$\begin{cases} 18(v'_A)_x + 6.6(v'_B)_x = 3.6 \\ -(v'_A)_x + (v'_B)_x = 0.19 \end{cases} \Rightarrow \begin{cases} (v'_A)_x = 0.0954 \ \frac{m}{s} & * \\ (v'_B)_x = 0.2854 \ \frac{m}{s} & * \end{cases} \quad 3$$

So the velocities of the masses after the impact are

$$\begin{cases} v'_A = (0.0954\,\hat{i} + 0.3\,\hat{j} - 0.02\,\hat{k}) \left(\frac{m}{s}\right) \\ v'_B = 0.2854\,\hat{i} \quad \left(\frac{m}{s}\right) \end{cases}$$