ج سوال ۱

ذکر تمام جزئیات زیر الزامی است و عدم ذکر جزئیات منجر به کسر نمره می شود. جواب ها نیز باید درست باشند.

A goes to B $\xrightarrow{+}$

$$\sum mv_1 = \sum mv_2 \Rightarrow \begin{cases} m_A v_A = (m_t + m_B) v_B \\ m_A s_A = (m_t + m_B) s_B \end{cases} \Rightarrow$$

Assume B moves x to the left, then A moves $(L-x)$ to the right

$$m_A(L-x) = (m_t + m_B)x \Rightarrow \boxed{x = \frac{m_A L}{m_A + m_B + m_t}}$$

$$x = \frac{36.2874 \times 1.2192}{36.2874 + 29.4835 + 9.0718} = 0.59113 \text{ m}$$

$\boxed{x = 0.59113 \text{m}}$

A and B go to other end

$$\sum mv_1 = \sum mv_2 \Rightarrow \begin{cases} (m_A + m_B) v = m_t v_t \\ (m_A + m_B) s = m_t s_t \end{cases}$$

Assume the toboggan moves x' to the right, the A and B move $(L - x')$ to the left

$$(m_A + m_B)(L - x') = m_t x' \Rightarrow x' = \frac{(m_A + m_B) L}{m_A + m_B + m_t} \Rightarrow$$

$$x' = \frac{(36.2874 + 29.4835) \times 1.2192}{36.2874 + 29.4835 + 9.0718} = 1.07142 \text{ m}$$

$\boxed{x' = 1.071418873 \text{m}}$

Net movement of sled is

$$\Delta = 1.07142 - 0.59113 = 0.4802888$$

$\boxed{\Delta = 0.4803 \text{ m}}$

جواب سوال دوم - روش اول

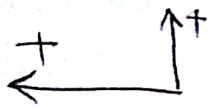
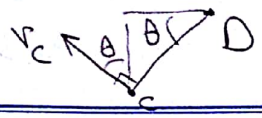
$$x = 2L\cos\theta \Rightarrow \dot{x} = -2L\dot{\theta}\sin\theta \Rightarrow \ddot{x} = -2L[\ddot{\theta}\sin\theta + \dot{\theta}^2\cos\theta]$$

$$\theta = 60^\circ \;,\; \dot{x} = 10\ \frac{m}{s} \;,\; \ddot{x} = -16\ \frac{m}{s^2}$$

$$\dot{x} = -2L\dot{\theta}\sin\theta \Rightarrow \omega = \dot{\theta} = \frac{\dot{x}}{2L\sin\theta} = \frac{10}{2\times 0.3\sin 60} = -19.2450\ \frac{rad}{s}$$

$$\ddot{x} = -2L\ddot{\theta}\sin\theta + 2L\dot{\theta}^2\cos\theta \Rightarrow \ddot{\theta} = -\frac{\ddot{x} + 2L\dot{\theta}^2\cos\theta}{2L\sin\theta} \Rightarrow$$

$$\ddot{\theta} = -\frac{-16 + 2\times 0.3\times(-19.2450)^2\times\cos 60}{2\times 0.3\times\sin 60} = -183.04122\ \frac{rad}{s^2}$$

محل مهر آموزش

$$\vec{v}_B = \vec{v}_A = +10\hat{i}\ \frac{in}{s^2} \Rightarrow v_{A/B} \quad \vec{v}_C = v_C(+\sin\theta\,\hat{i} + \cos\theta\,\hat{j})$$

$$\vec{v}_C = \vec{v}_B + \vec{v}_{C/B} \quad \vec{v}_{C/B} = v_{C/B}(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j})$$

$$+v_C\sin\theta\,\hat{i} + v_C\cos\theta\,\hat{j} = +v_A\hat{i} + (-v_{C/B}\sin\theta\,\hat{i} + v_{C/B}\cos\theta\,\hat{j}) \Rightarrow$$

$$\hat{i}: -v_C\sin\theta = +v_A + v_{C/B}\sin\theta$$

$$\hat{j}: v_C\cos\theta = v_{C/B}\cos\theta \Rightarrow v_C = v_{C/B}$$

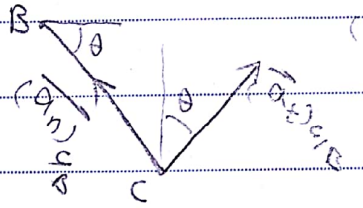
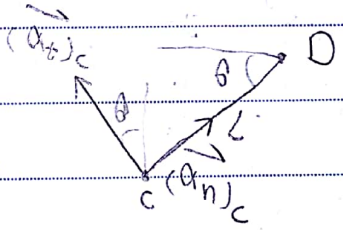
$$+2v_C\sin\theta = v_A \Rightarrow$$

$$v_C = v_{C/B} = \frac{v_A}{2\sin\theta} \Rightarrow v_C = v_{C/B} = \frac{10}{2\sin 60} = 5.773502692\ \frac{m}{s}$$

$$\omega_{BC} = \frac{v_{C/B}}{L} = \frac{5.7735}{0.3} = 19.24500897\ \frac{rad}{s} \Rightarrow \omega_{BC} = 19.245\ \frac{rad}{s}$$

$$\vec{v}_C = 5.7735[-\sin\theta\,\hat{i} + \cos\theta\,\hat{j}] \xrightarrow{\theta=60^\circ} \vec{v}_C = -5\hat{i} + 2.88675\hat{j}$$

$$\vec{a}_C = \vec{a}_B + \vec{a}_{C/B} \quad \vec{a}_B = \vec{a}_A = -10\hat{i}\ \frac{in}{s^2}$$

$$\vec{a}_C = L\alpha_{CD}[\sin\theta\,\hat{i} + \cos\theta\,\hat{j}] + \frac{v_C^2}{L}[\cos\theta\,i + \sin\theta\,\hat{j}]$$

$$\vec{a}_{C/B} = L\alpha_{BC}[-\sin\theta\,\hat{i} + \cos\theta\,\hat{j}] + \frac{v_{C/B}^2}{L}[\cos\theta\,\hat{i} + \sin\theta\,\hat{j}]$$

$$L\alpha_{CD}[\sin\theta\,\hat{i} + \cos\theta\,\hat{j}] + \frac{v_C^2}{L}[-\cos\theta\,i + \sin\theta\,\hat{j}] = -a_A\hat{i}$$

$$+L\alpha_{BC}[-\sin\theta\,\hat{i} + \cos\theta\,\hat{j}] + \frac{v_{C/B}^2}{L}[\cos\theta\,\hat{i} + \sin\theta\,\hat{j}]$$

باسمه تعالی

وزارت علوم، تحقیقات و فناوری
دانشگاه هرمزگان

نام : .....
نام و نام خانوادگی : ..... رشته تحصیلی : .....
شماره دانشجوئی : ..... تاریخ امتحان : .....
نام استاد : ..... امتحان درس : .....

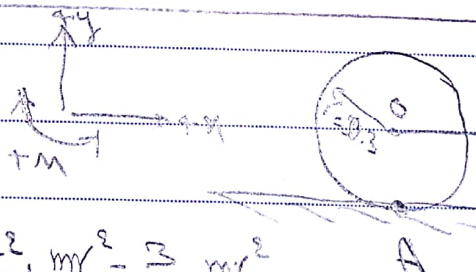
$$\begin{cases} l\alpha_{CD}\sin\theta - \dfrac{v_c^2}{l}\cos\theta = -a_A + l\alpha_{BC}\sin\theta + \dfrac{v_{c/B}^2}{l}\cos\theta \\ l\alpha_{CD}\cos\theta + \dfrac{v_c^2}{l}\sin\theta = l\alpha_{BC}\cos\theta + \dfrac{v_{c/B}^2}{l}\sin\theta \end{cases} \Rightarrow$$

$$\begin{cases} (l\sin\theta)\alpha_{CD} + (l\sin\theta)\alpha_{BC} = -a_A + (v_{c/B}^2 + v_c^2)\dfrac{\cos\theta}{l} \\ (l\cos\theta)\alpha_{CD} - (l\cos\theta)\alpha_{BC} = (-v_c^2 + v_{c/B}^2)\dfrac{\sin\theta}{l} \end{cases} \Rightarrow$$

$$\alpha_{BC} = \omega_{BC} = \ddot{\theta}_{BC} = 183.0414\ \frac{rad}{s^2} \qquad \boxed{\ddot{\theta}_{CD} = 183.0414\ \frac{rad}{s^2}}$$

جواب سوال ۳

$$\sum M)_A = I_A\alpha$$



$$\rightarrow P = 200\ N$$

$$I_A = I_0 + mr^2 = \frac{1}{2}mr^2 + mr^2 = \frac{3}{2}mr^2$$

$$-Pr = \left(\frac{3}{2}mr^2\right)\alpha \Rightarrow \alpha = -\frac{2Pr}{3mr^2} = -\frac{2P}{3mr} = -\frac{2\times200}{3\times100\times0.3} = -4.444\ \frac{rad}{s^2}$$

$$\boxed{\alpha = 4.444\ \frac{rad}{s^2}}$$

$$a = r\alpha = 0.3\times(-4.444) = -1.333\ \frac{m}{s^2}$$