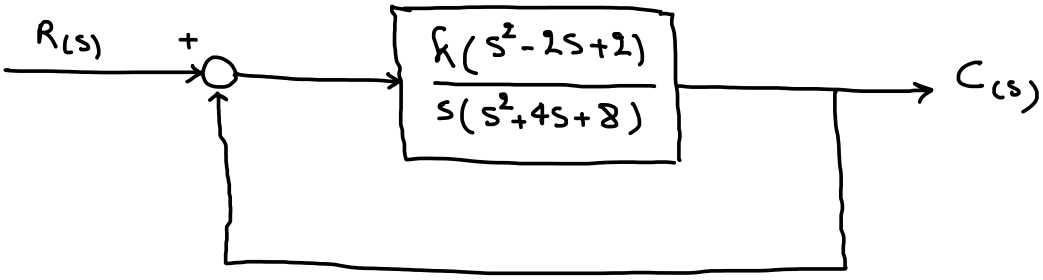
- consider the negative feedback system shown in figure:



Assume that the value of gain k is nonnegative ($k \geq 0$).

sketch the roo locus plot.

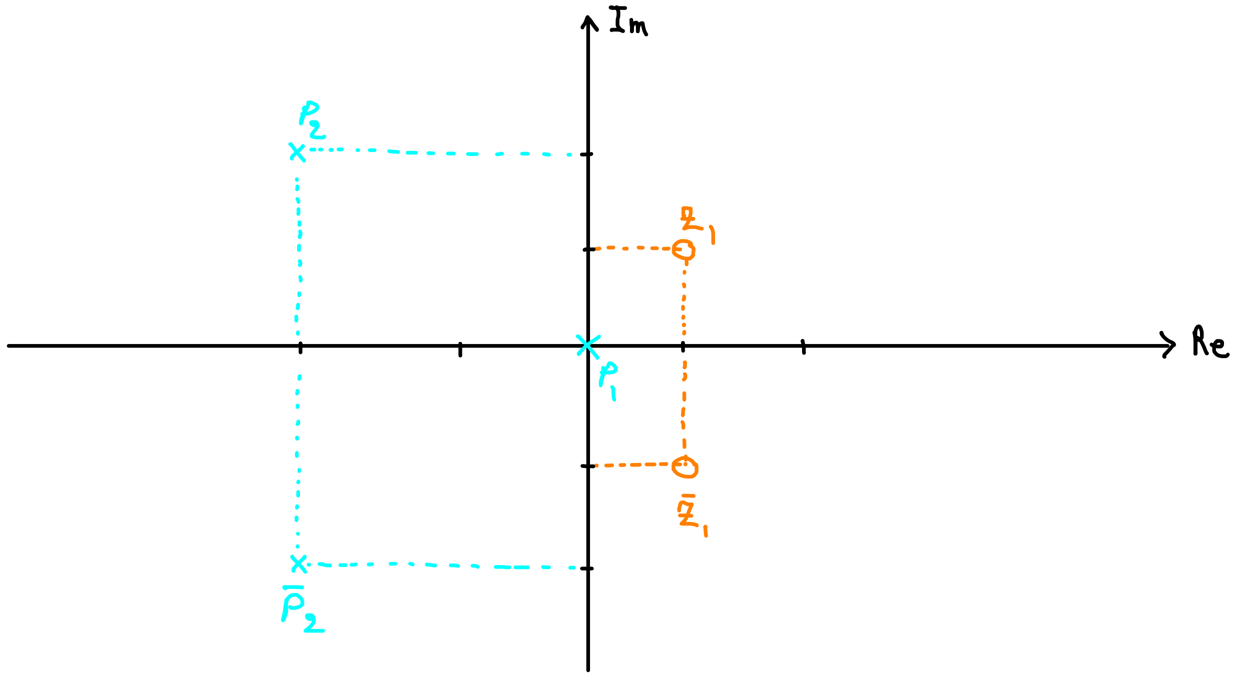
1

1.1 calculate zeros and poles of open-loop tranfer function.

Zeros: $s = \frac{-(-2) \pm \sqrt{(-2)^2 - 4\times1\times2}}{2\times1} = +1 \pm i$ or $+1 \pm j$ $\qquad i = j = \sqrt{-1}$

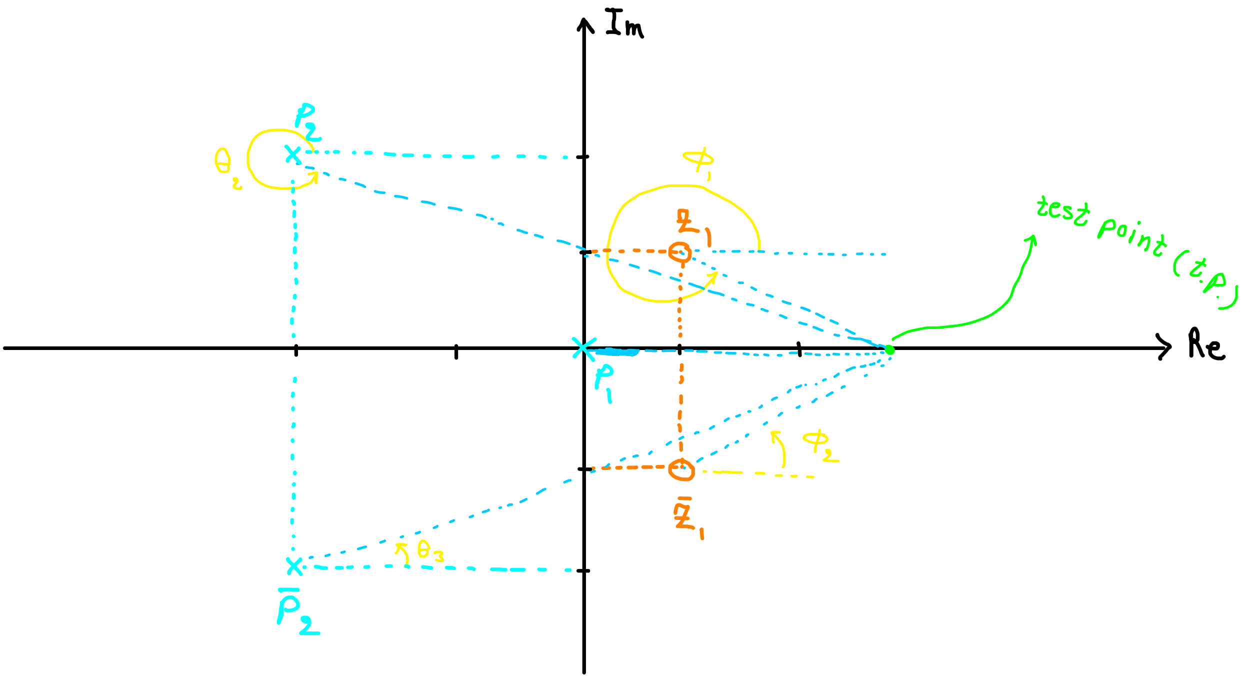
poles: $s = 0$ and $s = \frac{4 \pm \sqrt{4^2 - 4\times1\times8}}{2\times1} = -2 \pm 2i$

1.2 locate the zeros and poles of open-loop T.F. on the s plane.



2 [illegible]

2 Determine the root loci on the real axis.

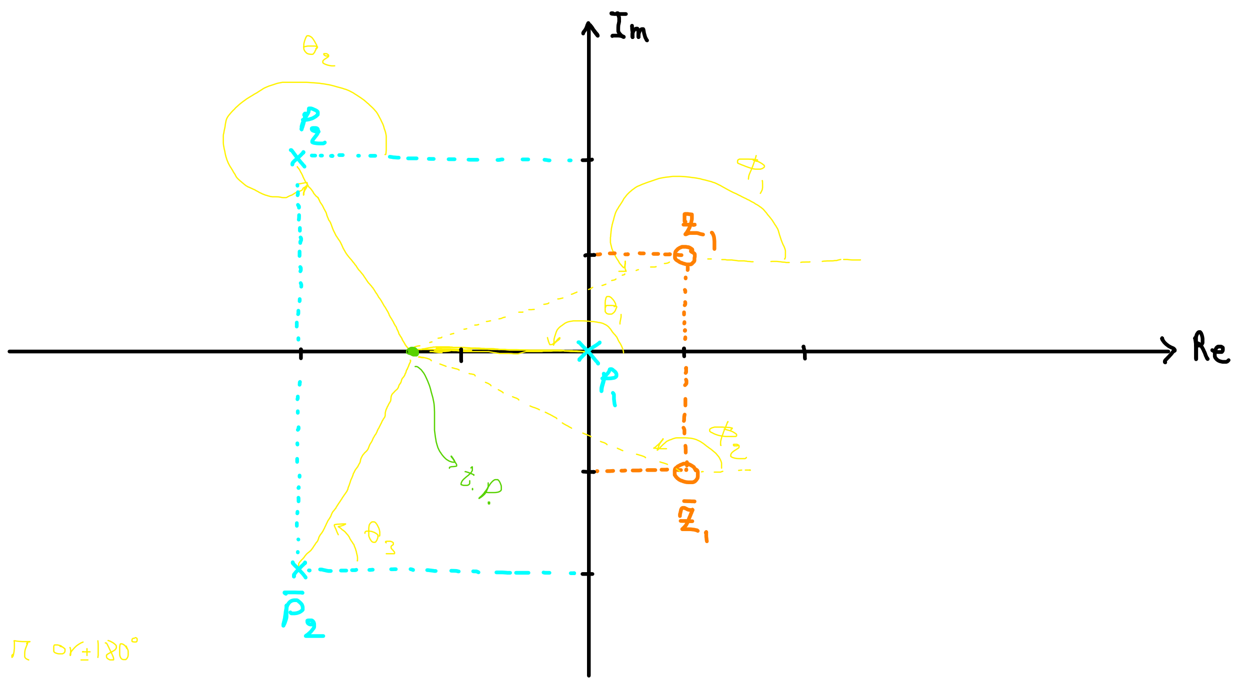


$\theta_1\ \angle P_1 = 0$, , $\phi_1 + \phi_2 = \angle z_1 + \angle \bar{z}_1 = 2\pi$, $\theta_2 + \theta_3 = \angle P_2 + \angle \bar{P}_2 = 2\pi$

Angle condition: $-\theta_1 - (\theta_2 + \theta_3) + (\phi_1 + \phi_2) = -\angle P_1 - (\angle P_2 + \angle \bar{P}_2) + (\angle z_1 + \angle \bar{z}_2) \overset{?}{=} \pm(2k-1)\pi$

$-0 - 2\pi + 2\pi \neq \pm(2k-1)\pi$ ⇒ The angle condition is not satisfied.

2.2



$\theta_1 = \angle P_1 = \pi$ or $\pm 180°$

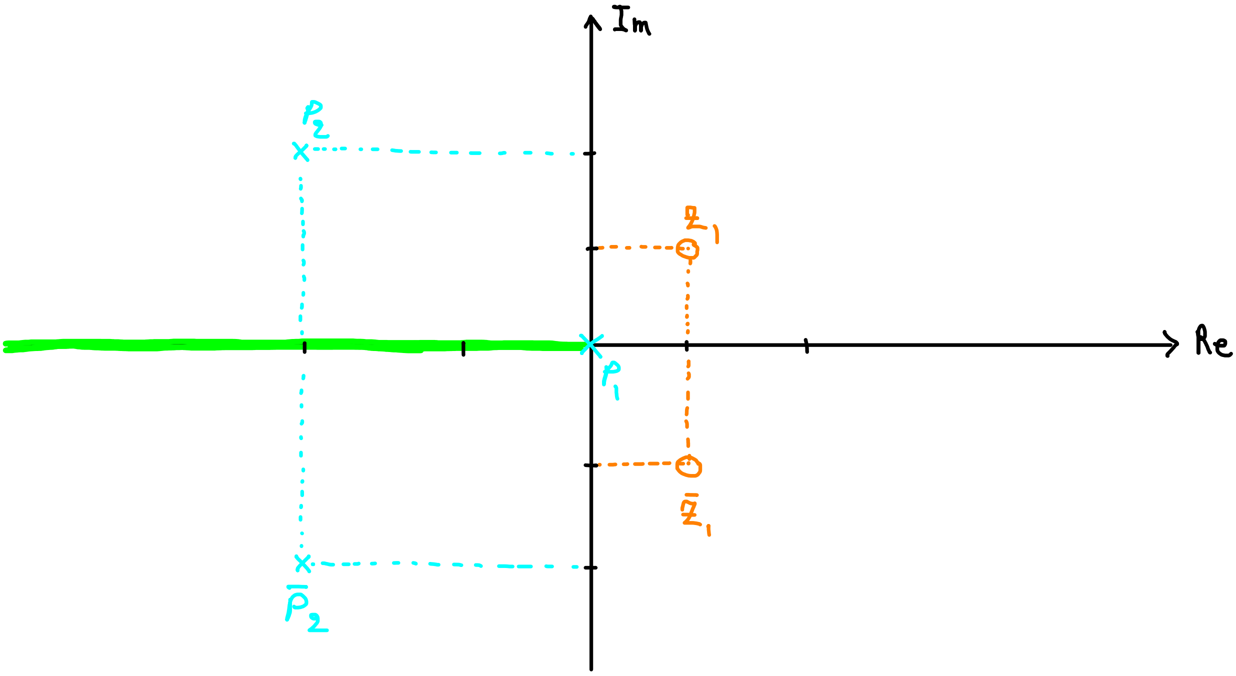
$\theta_2 + \theta_3 = \angle P_2 + \angle \bar{P}_2 = 2\pi$

$\varphi_1 + \varphi_2 = \angle z_1 + \angle \bar{z}_1 = 2\pi$

Angle condition: $-\theta_1 - (\theta_2 + \theta_3) + (\varphi_1 + \varphi_2) \overset{?}{=} \pm(2k-1)\pi \Rightarrow$

$$-\pi - (2\pi) + (2\pi) = -\pi = \pm(2k-1)\pi \quad , k=0$$

The angle condition is satisfied.



3 Determine the asymptots of the roo loci

3.1 Determine number of asymptots.

$$N = n - m = 3 - 2 = 1 \qquad \boxed{N=1}$$

3.2 Determine the angle of asymptots.

$$\varphi_A = \frac{\pm 180°(2k-1)}{n-m} = \frac{\pm 180°(2k-1)}{3-2} = \pm 180° \, (2k-1) \quad , k=0 \Rightarrow \varphi_A = \pm 180°$$

3.3 The asymptots intersect at a point on the real axis.

$$\sigma_A = \frac{\sum Re(P_i) - \sum Re(z_i)}{n-m} = \frac{(0-2-2)-(1+1)}{3-2} = -6$$

4 Find the breakaway and break-in points.

4.1 Write the characteristic equation.

$$1 + \frac{k(s^2-2s+2)}{s(s^2+4s+8)} = 0 \qquad (1)$$

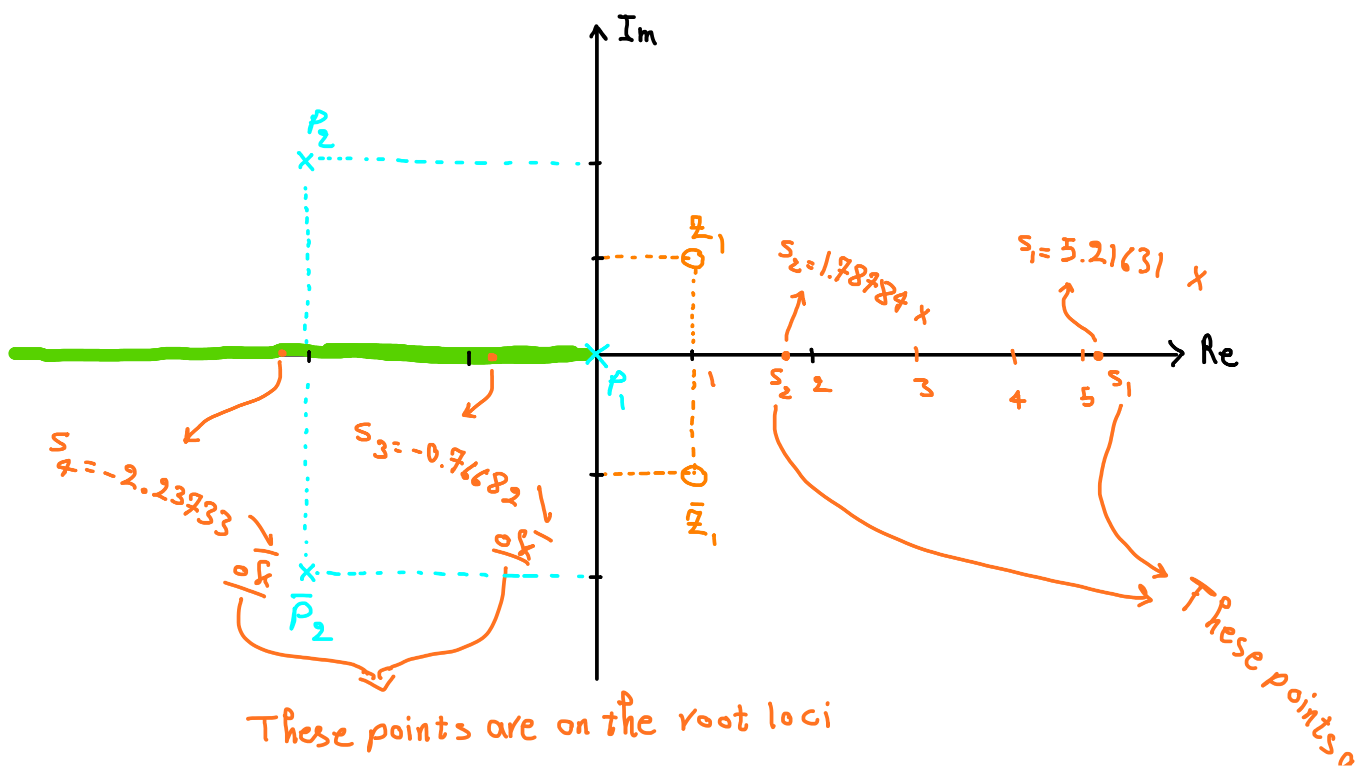
4.2 Obtain k frome the characteristic equation.

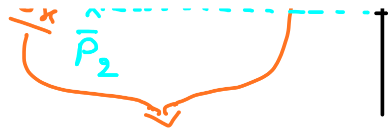
$$(1) \Rightarrow \frac{k(s^2-2s+2)}{s(s^2+4s+8)} = -1 \Rightarrow k = \frac{s(s^2+4s+8)}{s^2-2s+2} = \frac{s^3+4s^2+8s}{s^2-2s+2} \qquad (2)$$

4.3 Determine s from $\frac{dk}{ds} = 0$

$$(2) \Rightarrow \frac{dk}{ds} = \frac{(3s^2+8s+8)(s^2-2s+2) - (2s-2)(s^3+4s^2+8s)}{(s^2-2s+2)^2} = \frac{s^4-4s^3-10s^2+16s+16}{(s^2-2s+2)^2}$$

$$\frac{dk}{ds} = 0 \Rightarrow s^4-4s^3-10s^2+16s+16 = 0 \Rightarrow \begin{cases} s_1 = 5.21631 \\ s_2 = 1.78784 \\ s_3 = -0.76682 \\ s_4 = -2.23733 \end{cases}$$





These points are on the root loci

These points are not on the root loci

- Calculate k for S_1, S_2, S_3 and S_4:

- $S_1 = 5.21631 \overset{(2)}{\Longrightarrow} k\Big|_{s=s_1} = \dfrac{s_1^3 + 4s_1^2 + 8s_1}{s_1^2 - 2s_1 + 2} = -15.5776 < 0$ ✗

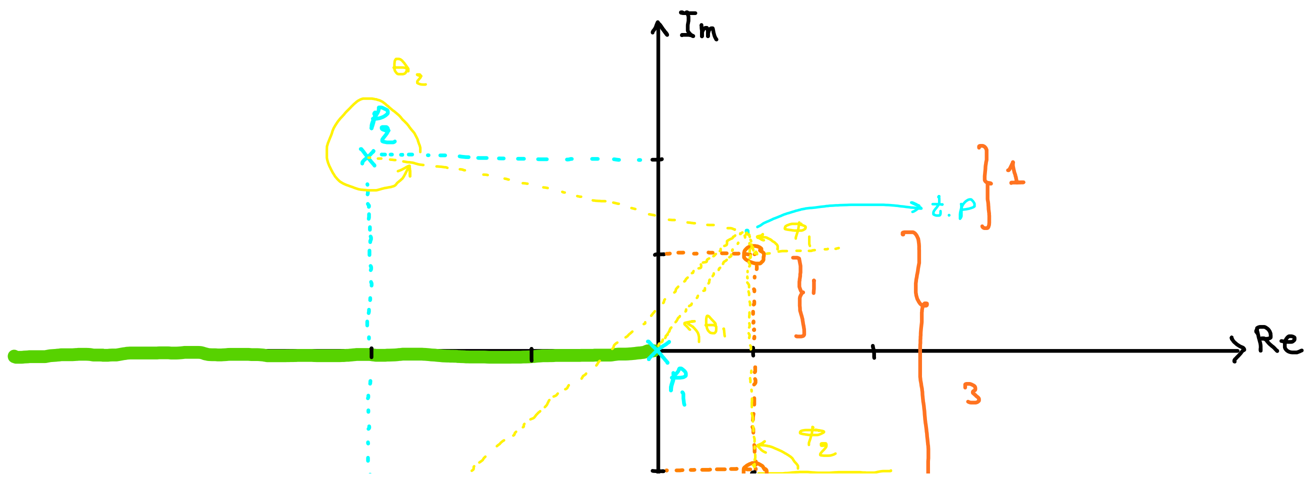
- $S_2 = 1.78784 \overset{(2)}{\Longrightarrow} k\Big|_{s=s_2} = \dfrac{s_2^3 + 4s_2^2 + 8s_2}{s_2^2 - 2s_2 + 2} = -20.2400 < 0$ ✗

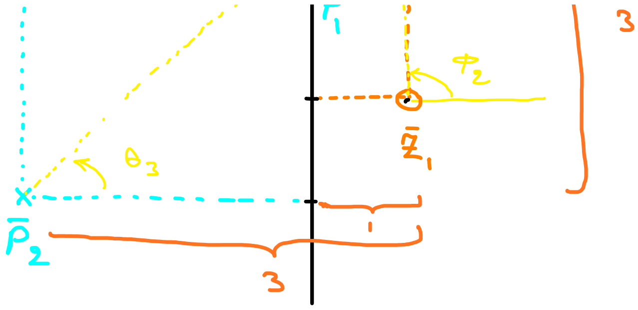
- $S_3 = -0.76682 \overset{(2)}{\Longrightarrow} k\Big|_{s=s_3} = \dfrac{s_3^3 + 4s_3^2 + 8s_3}{s_3^2 - 2s_3 + 2} = 1.0271 \geq 0 \searrow$ ok

- $S_4 = -2.23733 \overset{(2)}{\Longrightarrow} k\Big|_{s=s_4} = \dfrac{s_4^3 + 4s_4^2 + 8s_4}{s_4^2 - 2s_4 + 2} = 0.7905 \geq 0 \searrow$ ok

5 Determine the angle of departure (angle of arrival) of the root locus from a complex poles (at a complex zeros).

5.1





$\Phi_1 = ?$ $\Phi_2 \simeq 90^\circ$ $\theta_3 \simeq tg^{-1}(\frac{3}{3}) = 45^\circ$, $\theta_2 \simeq 360^\circ - tg^{-1}(\frac{1}{3}) = 360 - 18.435 = 341.565^\circ$

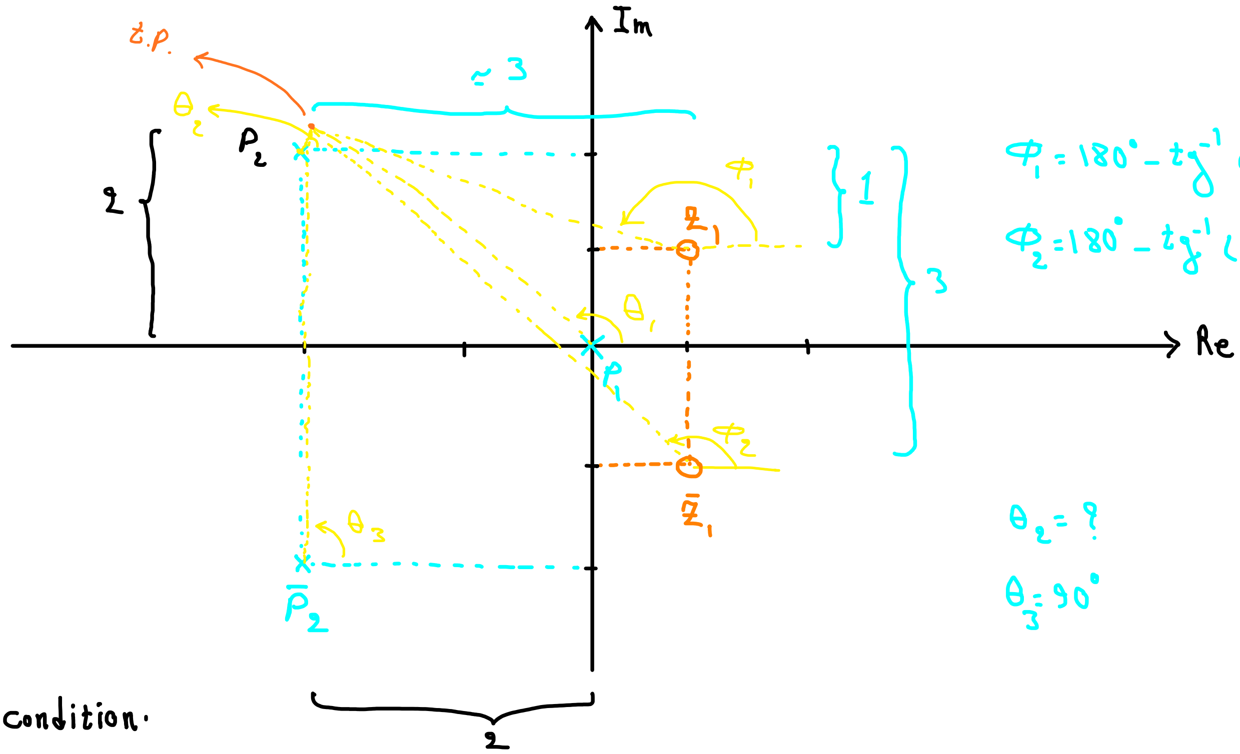
$\theta_1 = tg^{-1}(\frac{1}{1}) = 45^\circ$

Angle condition: $-\theta_1 - \theta_2 - \theta_3 + \Phi_1 + \Phi_2 = \pm(2k-1)180^\circ \Rightarrow$

$-45^\circ - 341.565^\circ - 45^\circ + \Phi_1 + 90^\circ = \pm(2k-1)180^\circ \Rightarrow \Phi_1 = \pm(2k-1)180^\circ + 341.565^\circ \Rightarrow$

$k=0 \Rightarrow \mp 180^\circ + 341.565^\circ = \Phi_1 \Rightarrow$ $\boxed{\Phi_1 = 161.565^\circ}$

5.2



$\Phi_1 = 180^\circ - tg^{-1}(\frac{1}{3})$

$\Phi_2 = 180^\circ - tg^{-1}(\frac{3}{3})$

$\theta_2 = ?$

$\theta_3 = 90^\circ$

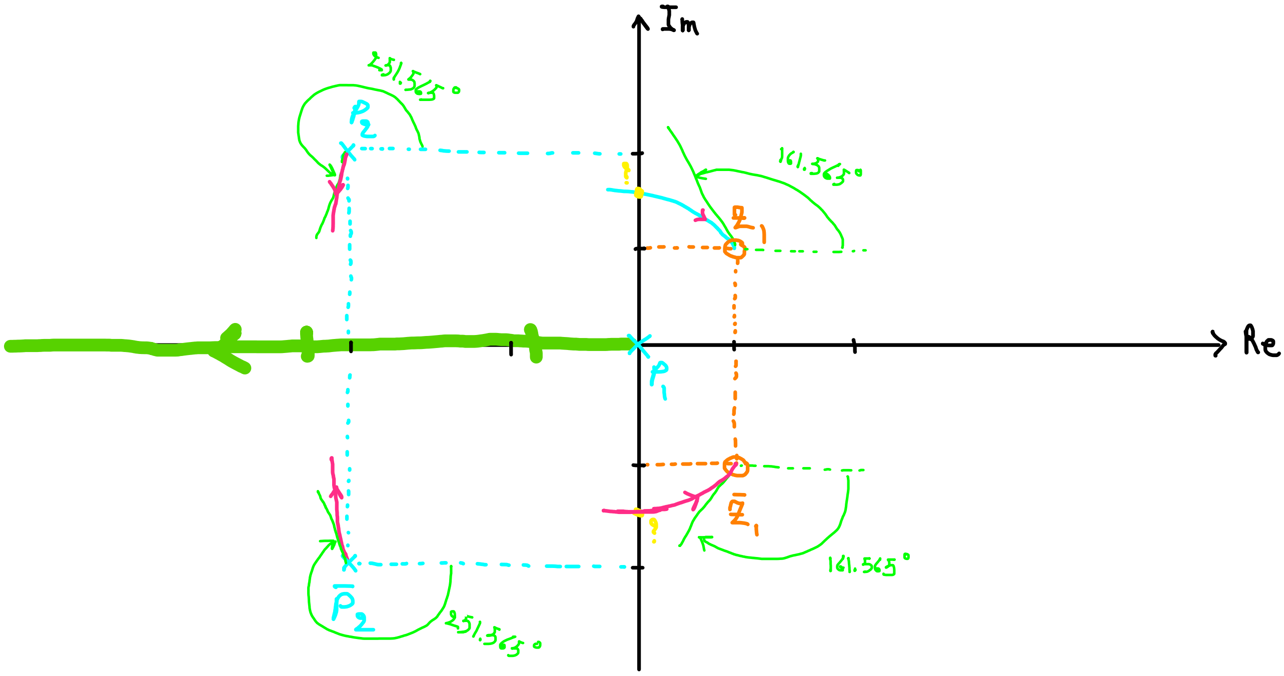
Angle condition.

$\theta_1 = 180^\circ - tg^{-1}(\frac{2}{2}) = 135^\circ$, $\theta_2 = 90$, $\theta_3 = ?$, $\Phi_1 = 180^\circ - tg^{-1}(\frac{1}{3}) = 161.565^\circ$

$\Phi_2 = 180^\circ - tg^{-1}(\frac{3}{3}) = 135^\circ$

$$-\theta_1-\theta_2-\theta_3+\varphi_1+\varphi_2=\pm(2k-1)180^\circ \Rightarrow -135^\circ-\theta_2-90^\circ+161.565^\circ+135^\circ=\pm180^\circ(2k-1)$$

$$\Rightarrow \theta_2=71.565^\circ \mp (2k-1)180^\circ \Rightarrow \boxed{\theta_2=251.565^\circ}$$



6 Find the points where the root loci cross the imaginary axis.

6.1 Routh's stability criterion

$$(1) \Rightarrow 1+\frac{k(s^2-2s+2)}{s(s^2+4s+8)}=0 \Rightarrow \frac{s^3+4s^2+8s+k(s^2-2s+2)}{s(s^2+4s+8)}=0 \Rightarrow$$

$$s^3+(4+k)s^2+(8-2k)s+2k=0$$

s^3	1	$(8-2k)$
s^2	$(4+k)$	$2k$
s^1	$\dfrac{(4+k)(8-2k)-1\times 2k}{(4+k)}$	0
s^0	$2k$	

$s^0 \quad 2k$

$$32-8k+8k-2k^2-2k=0 \Rightarrow k^2+k-16=0 \Rightarrow$$

$$k=\frac{-1\pm\sqrt{1^2-4\times1\times(-16)}}{2\times1}=\begin{cases} k=3.5311 \geq 0 \quad ✓ \\ k=-4.5311<0 \quad \times \end{cases}$$

Auxiliary equation: $(4+k)s^2+2k=0$, $k=3.5311 \Rightarrow$

$$7.5311s^2+7.0622=0 \Rightarrow s=\pm\sqrt{\frac{-7.0622}{7.5311}}=\pm 0.968i$$

$$\boxed{s=\pm 0.968i}$$

6.2 Leteing $s=i\omega=j\omega$ $(i=j=\sqrt{-1})$ in the characteristic equation.

$$s^3+(4+k)s^2+(8-2k)s+2k=0 \xrightarrow{s=i\omega}$$

$$(i\omega)^3+(4+k)(i\omega)^2+(8-2k)(i\omega)+2k=0 \Rightarrow$$

$$\underbrace{-i\omega^3}_{Im}\ \underbrace{-(4+k)\omega^2}_{Re}\ \underbrace{+(8-2k)(i\omega)}_{Im}\ \underbrace{+2k}_{Re}=0 \Rightarrow$$

$$2k-(4+k)\omega^2+i\left[-\omega^3+(8-2k)\omega\right]=0 \Rightarrow \begin{cases} 2k-4\omega^2-k\omega^2=0 & (*) \\ 8\omega-2k\omega-\omega^3=0 \text{ or } 8-2k-\omega^2=0 & (**) \end{cases}$$

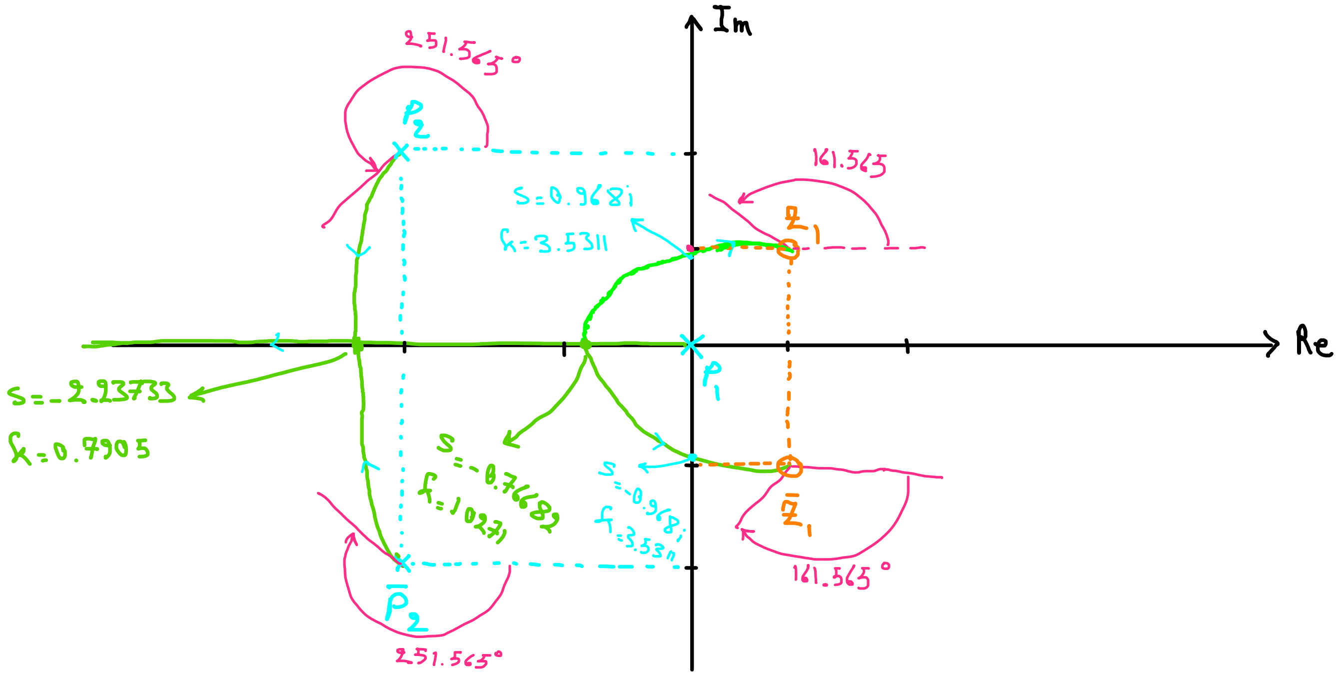
$(**) \Rightarrow \omega^2=8-2k \quad (***)$

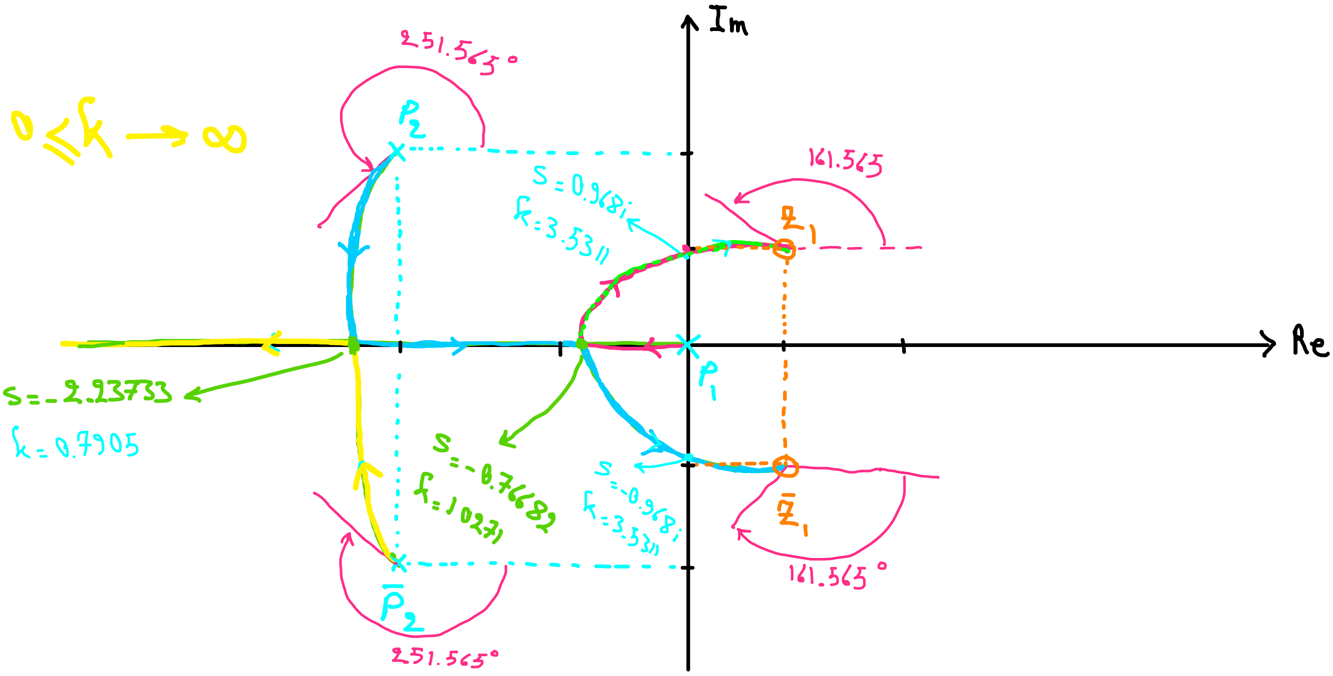
$(***)$ into $(*) \Rightarrow 2k-(4+k)(8-2k)=0 \Rightarrow 2k^2+2k-32=0$ or $\underline{k^2+k-16=0}$,

$$k=\frac{-1\pm\sqrt{1^2-4\times1\times(-16)}}{2\times1} \Rightarrow \begin{cases} k=3.5311>0 \ ✓ \\ k=-4.5311<0 \ \times \end{cases}$$

$$(***) \Rightarrow \omega=\pm\sqrt{8-2k} \Rightarrow \omega=\pm\sqrt{8-2\times3.5311}=\pm 0.968 \Rightarrow s=\pm 0.968i$$

$(***) \Rightarrow \omega = \pm\sqrt{8-2k} \Rightarrow \omega = \pm\sqrt{8-2\times 3.5311} = \pm 0.968 \Rightarrow s = \pm 0.968i$



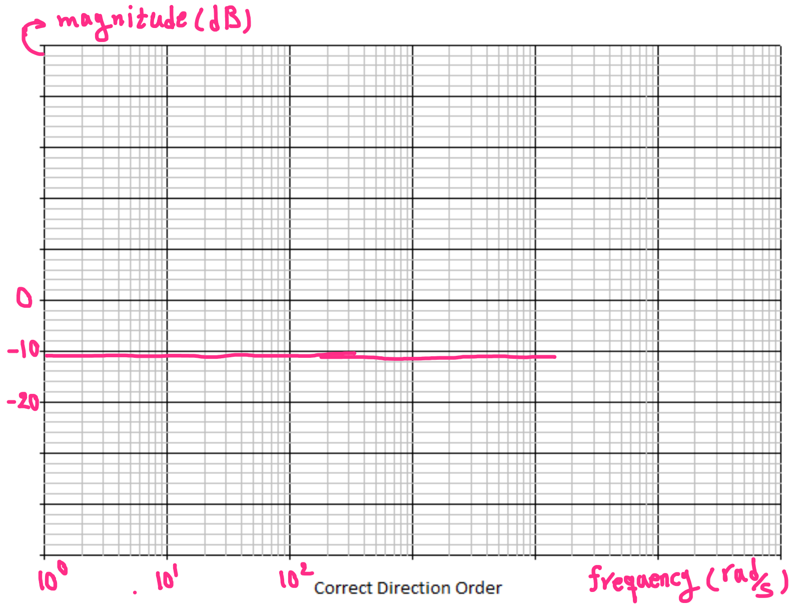


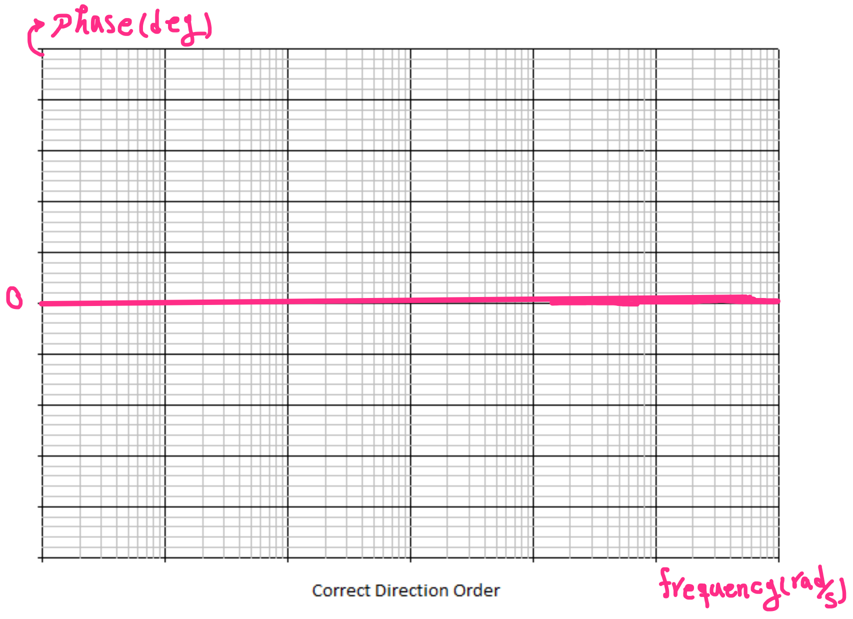
Example: Sketch bode diagram of $G_{(j\omega)} = \frac{k(s^2+2s+2)}{\ldots}$, $k=1$

Example : Sketch bode diagram of $G_{(j\omega)} = \frac{K(s^2+2s+2)}{s(s^2+4s+8)}$, $K=1$

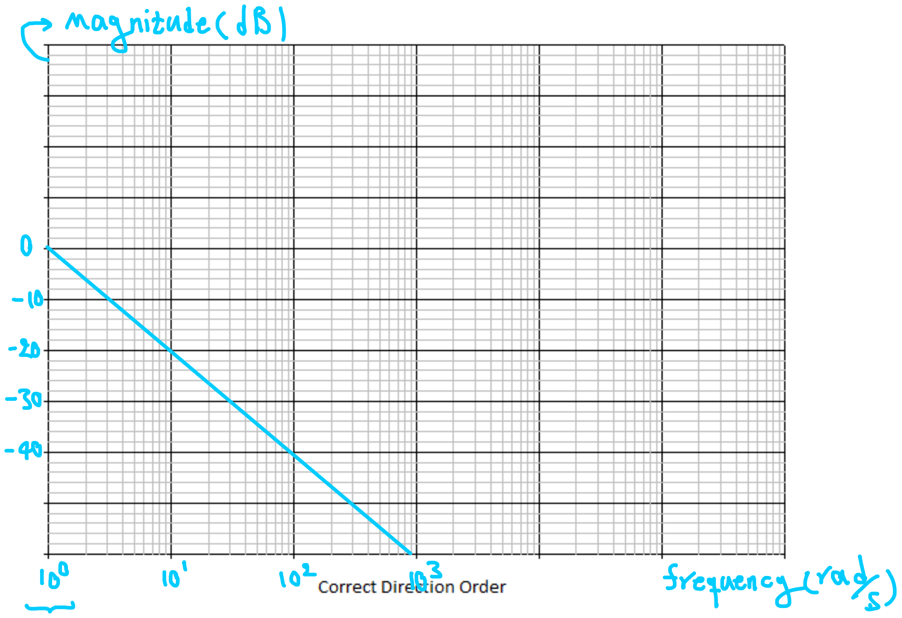
Write the fraction as standard $G_{(j\omega)} = \frac{2(\frac{s^2}{2}+s+1)}{8(\frac{s^2}{8}+\frac{s}{2}+1)} = \frac{2}{8}\frac{(1+s+\frac{s^2}{2})}{(1+\frac{s}{2}+\frac{s^2}{8})}$

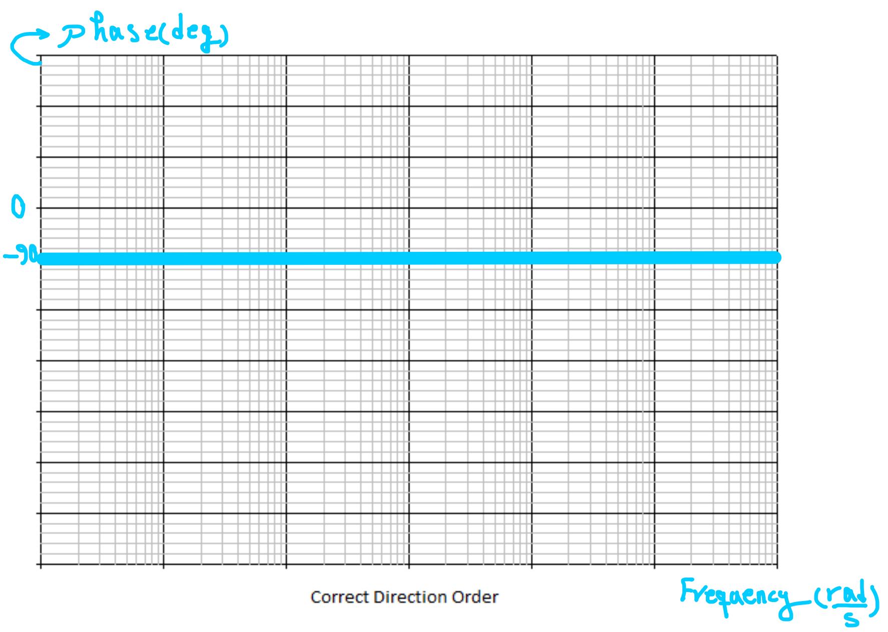
1. The first component $= \frac{2}{8}$ or $\frac{1}{4}$ $\Rightarrow 20 \log(\frac{1}{4}) = -12.0412$





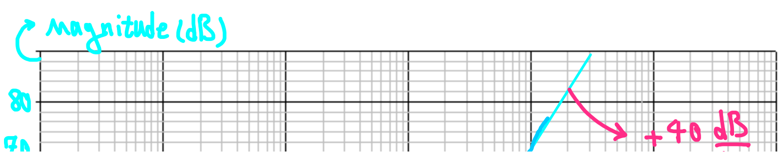
2: The second component: $\frac{1}{s}$

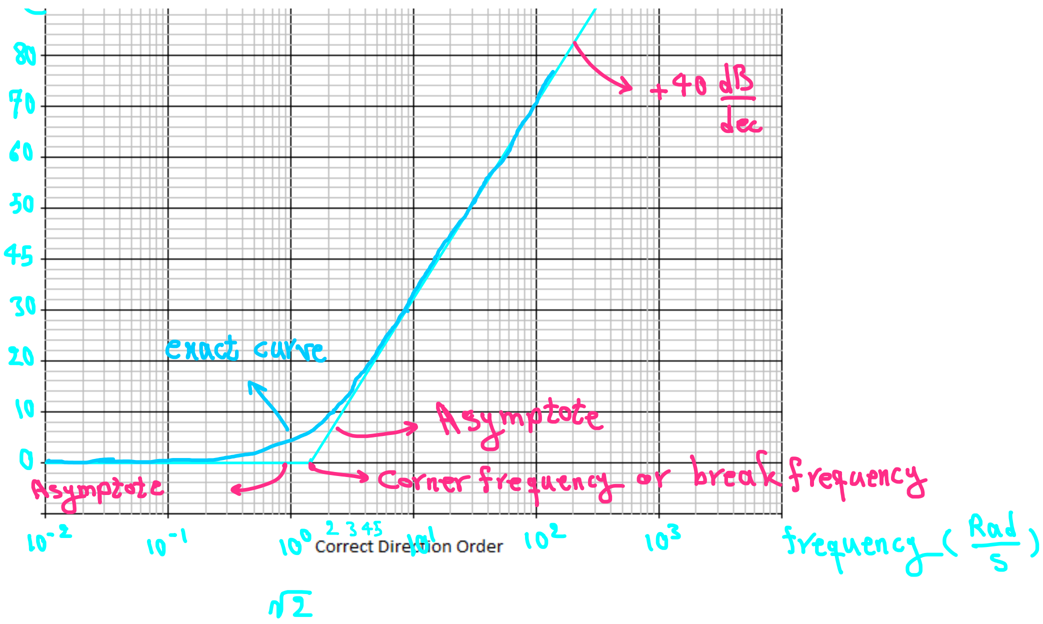


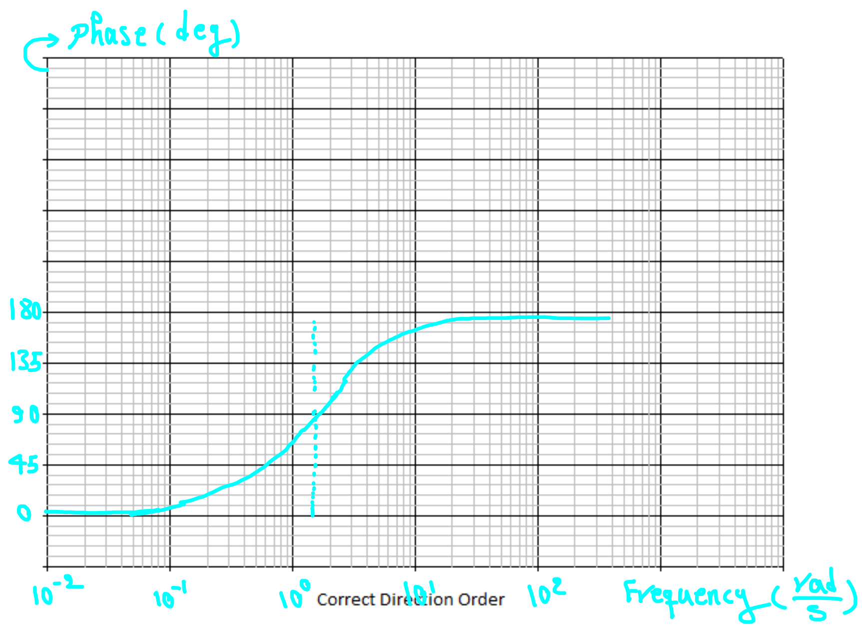


(3) The third component: $1 + s + \frac{s^2}{2}$:

corner frequency or break frequency $= \sqrt{2}$

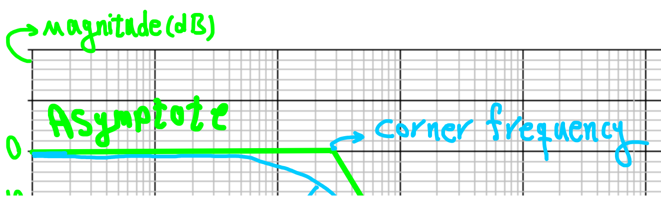




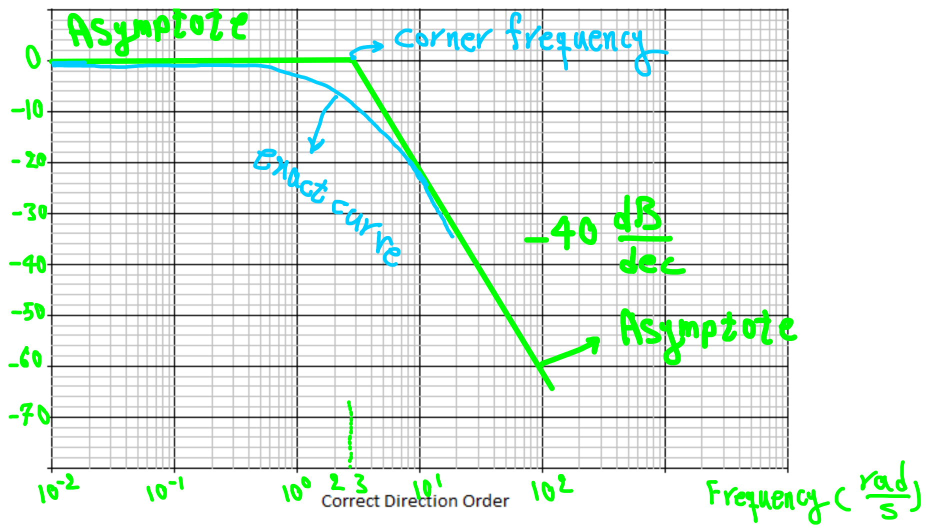


4: The fourth component: $\dfrac{1}{1+\frac{s}{2}+\frac{s^2}{8}}$

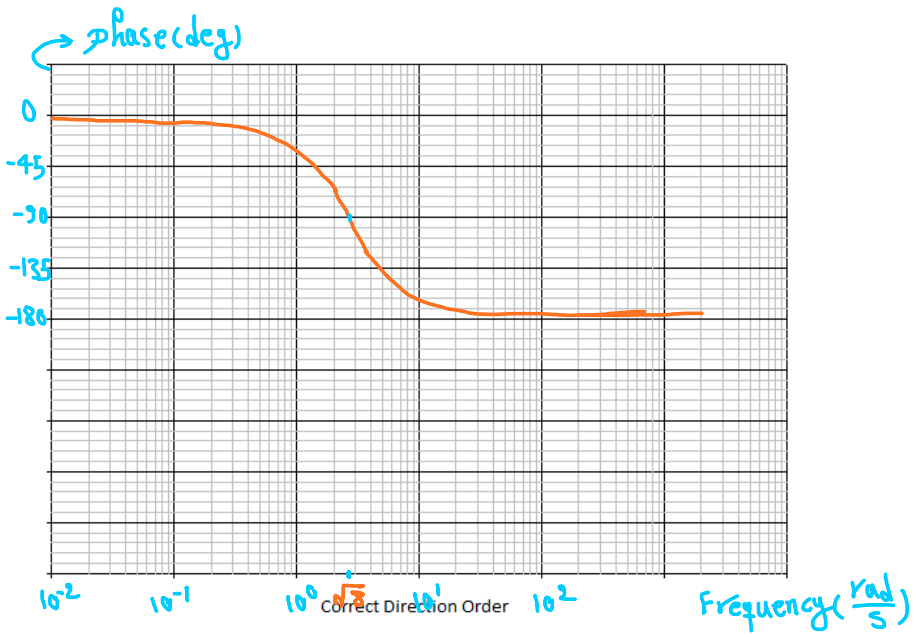
Corner frequency or Break frequency: $\sqrt{8} = 2\sqrt{2} = 2.8284$

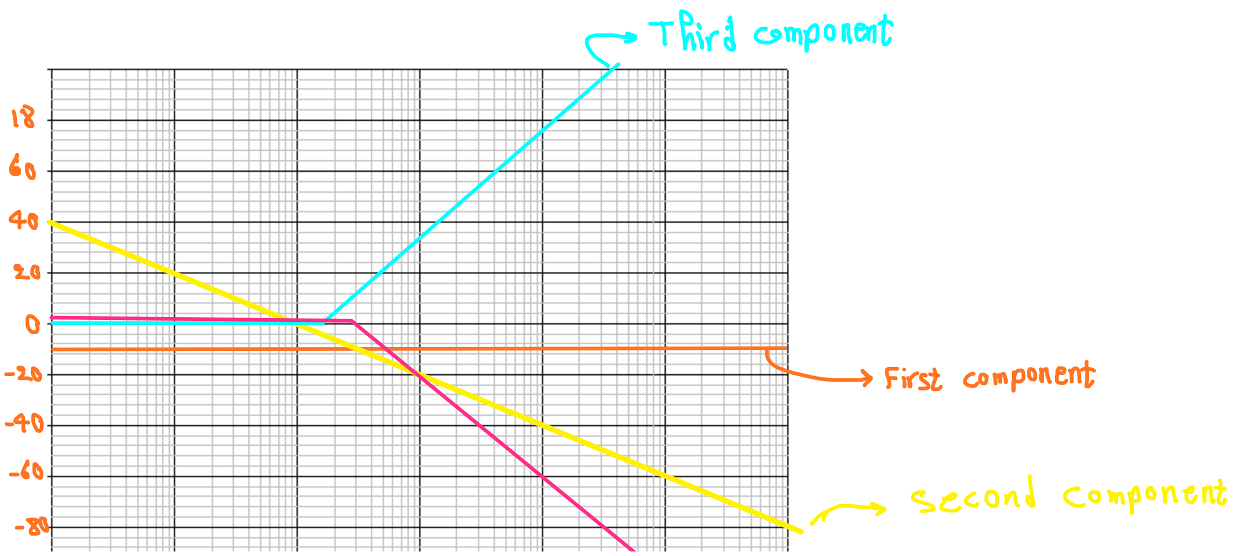


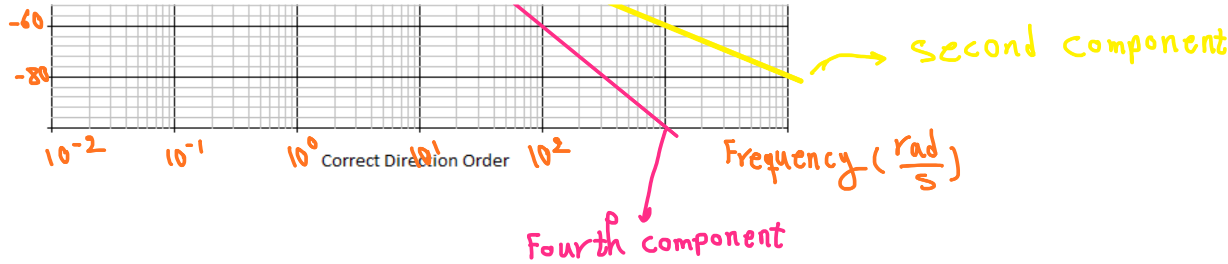
C.F. or B.F. = $2\sqrt{2}$

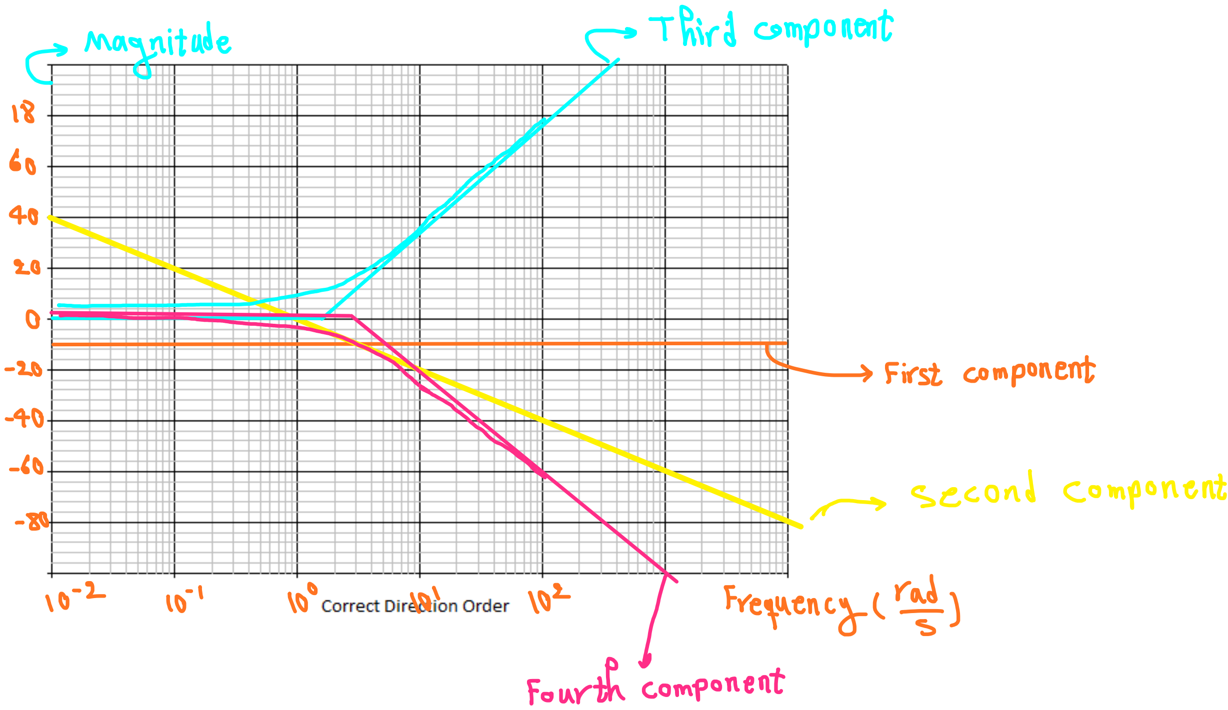


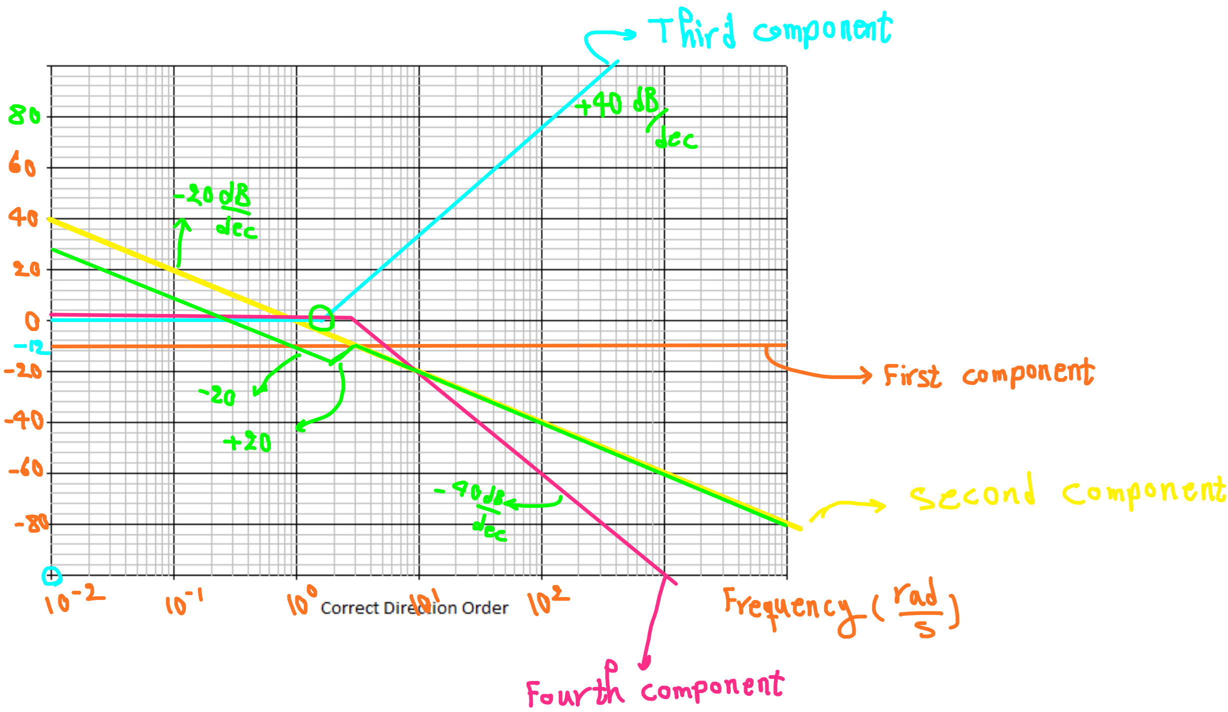
C.F. or B.F. = $2\sqrt{2}$

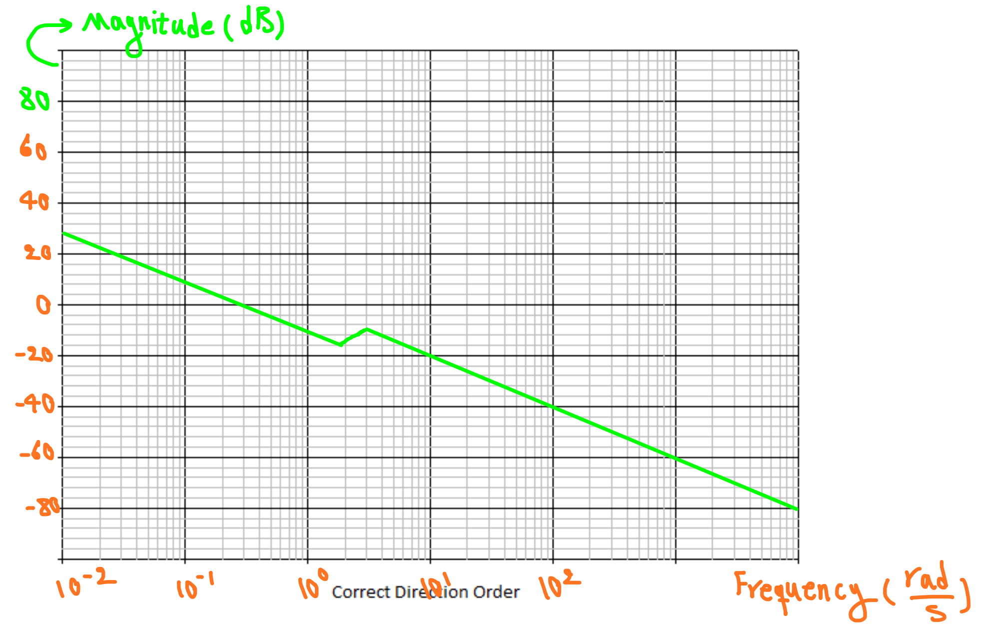


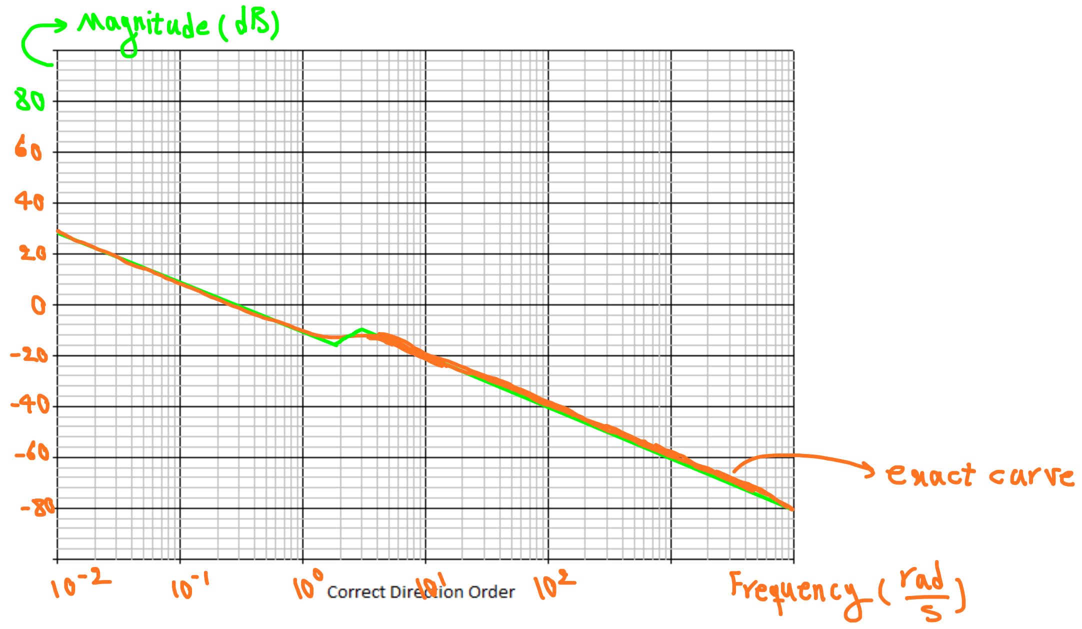


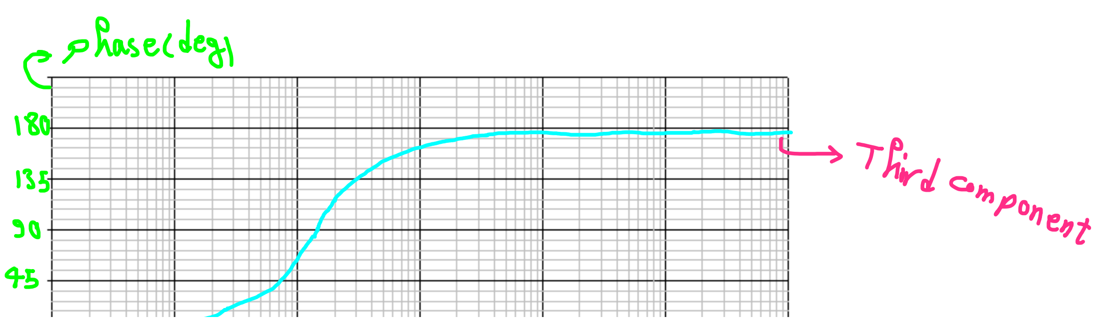

-60
-80
second component
Correct Direction Order
Frequency (rad/s)
Fourth component

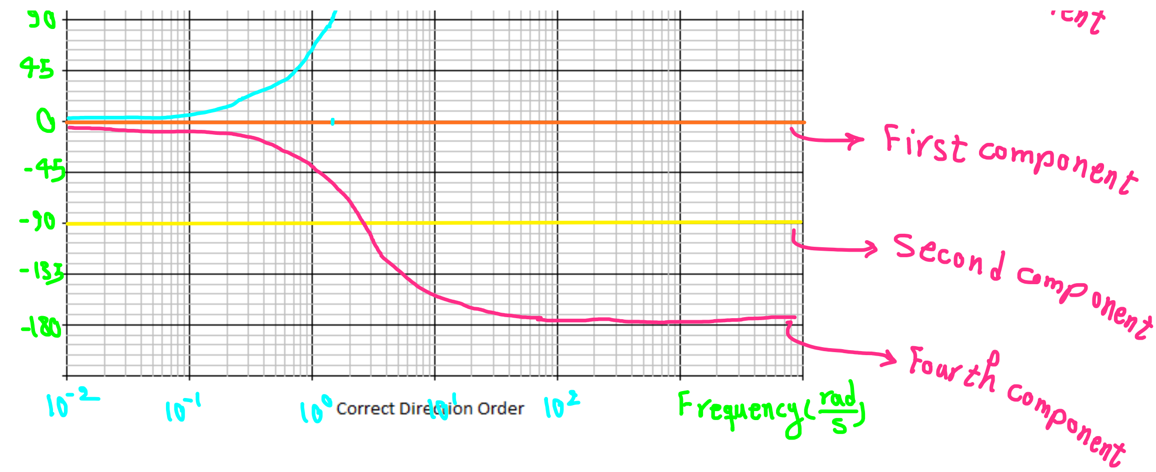

Magnitude
Third component
18
60
40
20
0
-20
-40
-60
-80
First component
second component
Correct Direction Order
Frequency (rad/s)
Fourth component

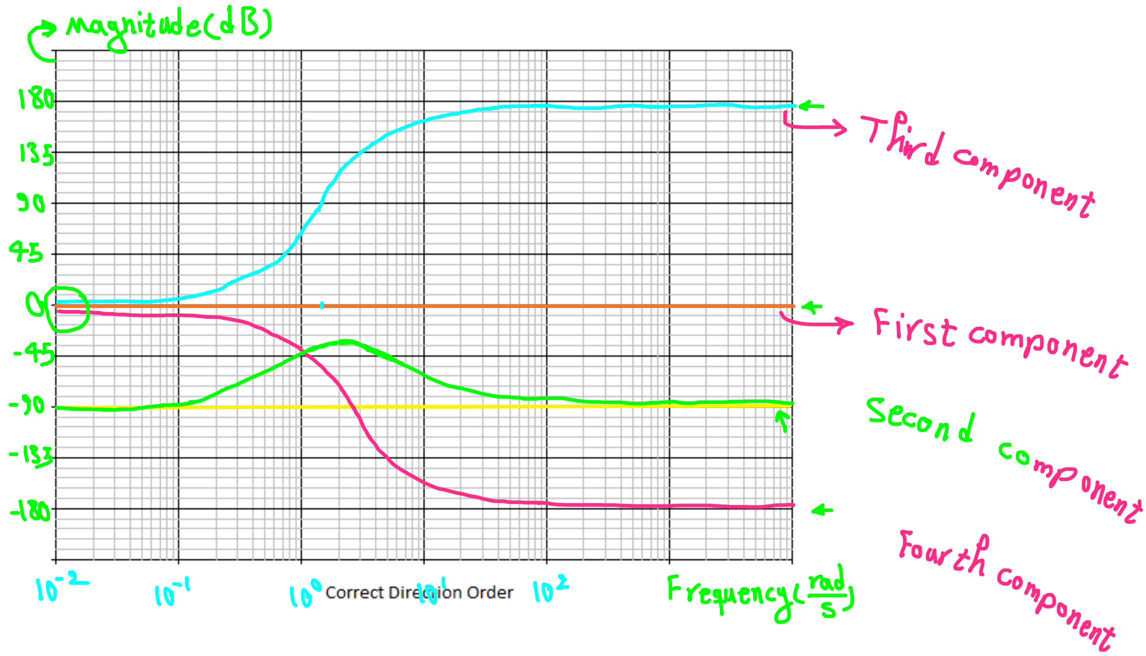

Third component
80
60
40
20
0
-12
-20
-40
-60
-80
+40 dB/dec
-20 dB/dec
-20
+20
-40 dB/dec
First component
second component
Correct Direction Order
Frequency (rad/s)
Fourth component

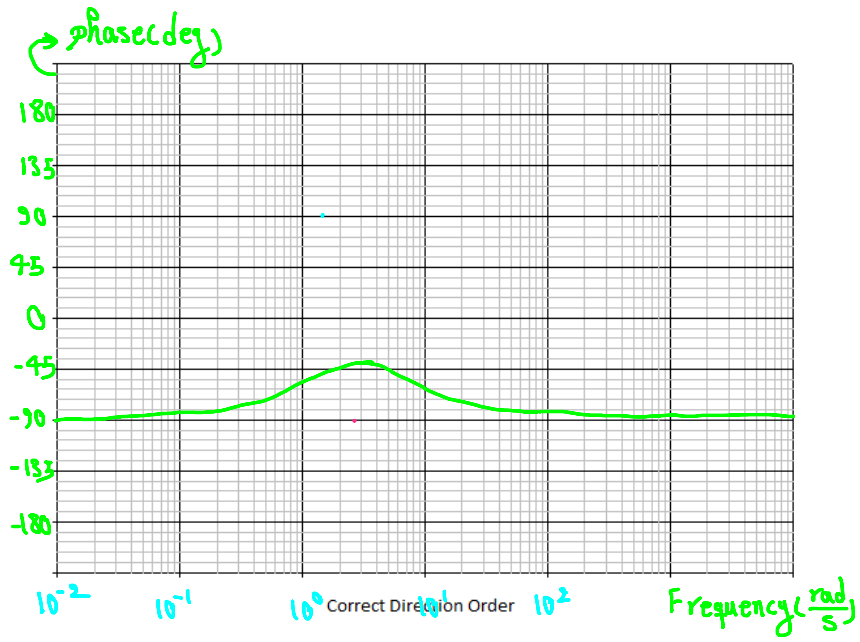

Magnitude (dB)
80
60
40
20
0
-20
-40
-60
-80
10^{-2}
10^{-1}
10^{0}
Correct Direction Order
10^{1}
10^{2}
Frequency (rad/s)


Magnitude (dB)
80
60
40
20
0
-20
-40
-60
-80
exact carve
10^{-2}
10^{-1}
10^{0}
Correct Direction Order
10^{1}
10^{2}
Frequency (rad/s)

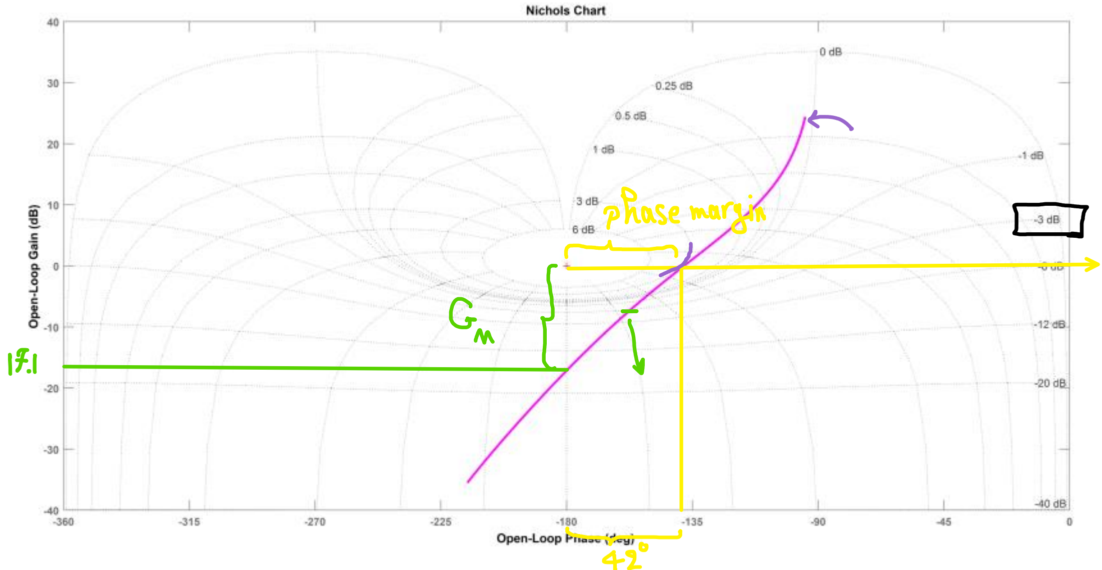

phase(deg)
180
135
90
45
Third component


90
45
0
-45
-90
-135
-180
Correct Direction Order
Frequency(rad/s)
First component
Second component
Fourth component


magnitude(dB)
180
135
90
45
0
-45
-90
-135
-180
Correct Direction Order
Frequency(rad/s)
Third component
First component
Second component
Fourth component


phase(deg)
180
135
90
45
0
-45
-90
-135
-180
Correct Direction Order
Frequency(rad/s)

Find Gain margin, Phase margin M_r, $\xi = ?$



$M_r \simeq 2.9$ dB or $20 \log M_r = 2.9 \Rightarrow M_r = 10^{\left(\frac{2.9}{20}\right)} \simeq 1.396$

$\xi = ?$

Damping ratio (ξ) cannot be calculated. Because the order of the system is not stated.

★ M_p is different from M_r