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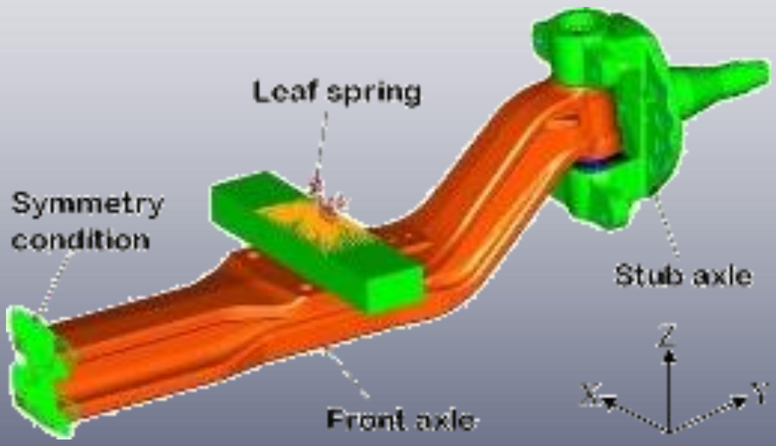
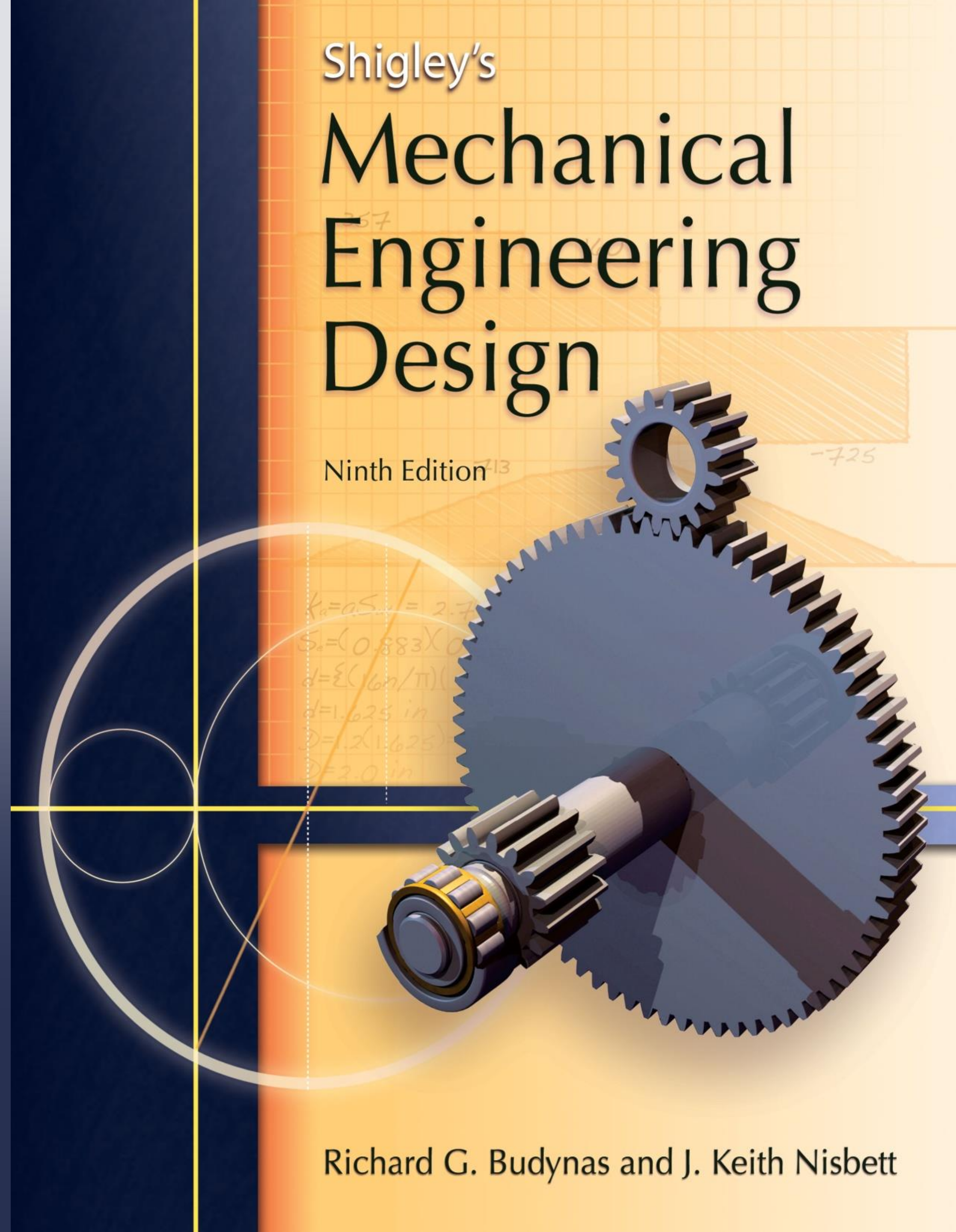


Chapter 3

Load and Stress Analysis

Faculty of Engineering
Mechanical Dept.



Chapter Outline

Equilibrium and Free-Body Diagram

System: used to denote any *isolated* part or portion of a machine or structure. A system may consist of

- a particle, or several particles,
- a part of rigid body, an entire rigid body, or several rigid bodies.

Equilibrium: A system is said to be in *equilibrium* if it is motionless or has a constant velocity, i.e.; zero acceleration. The phrase *static equilibrium* is also used to imply that the system is at rest. For equilibrium, the forces and moments acting on the system balance such that

$$\sum \vec{F} = \sum F_x \vec{i} + \sum F_y \vec{j} + \sum F_z \vec{k} = \vec{0} \quad \text{(Vectorial Representation)}$$

or

$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum F_z = 0 \end{cases} \quad \text{(Scalar Representation)}$$

and

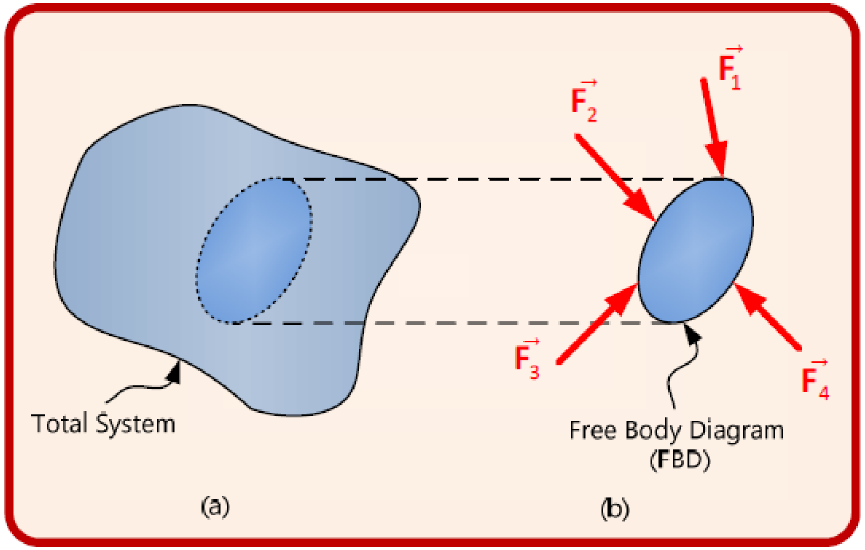
$$\sum \vec{M} = \sum M_x \vec{i} + \sum M_y \vec{j} + \sum M_z \vec{k} = \vec{0} \quad \text{(Vectorial Representation)}$$

or

$$\begin{cases} \sum M_x = 0 \\ \sum M_y = 0 \\ \sum M_z = 0 \end{cases} \quad \text{(Scalar Representation)}$$

Free-Body Diagram (FBD)

Free Body Diagram (FBD) is a sketch that shows a *body of interest* isolated from all interacting bodies. Once the *body of interest* is selected, the forces and moments exerted by *all* other bodies *on* the one being considered must be determined and shown in the diagram, see Figure



Free-Body Diagram

- The diagram establishes the directions of reference axes, provides a place to record the dimensions of the subsystem and the magnitudes and directions of the known forces, and helps in assuming the directions of unknown forces.
- The diagram simplifies your thinking because it provides a place to store one thought while proceeding to the next.
- The diagram provides a means of communicating your thoughts clearly and unambiguously to other people.
- Careful and complete construction of the diagram clarifies fuzzy thinking by bringing out various points that are not always apparent in the statement or in the geometry of the total problem. Thus, the diagram aids in understanding all facets of the problem.
- The diagram helps in the planning of a logical attack on the problem and in setting up the mathematical relations.
- The diagram helps in recording progress in the solution and in illustrating the methods used.
- The diagram allows others to follow your reasoning, showing *all* forces.

Example 3.1

Draw a free body diagram for the beam shown in Fig. 4-2a.

Solution

Referring to Figure 4-2 (b):

- Two concentrated forces P_1 and P_2 are applied to the beam.
- The weight of the beam is represented by the force W, which has a line of action that passes through the center of gravity G of the beam.

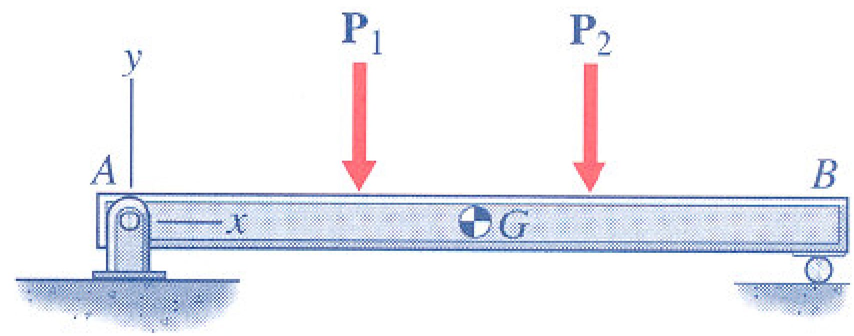


Figure (a)

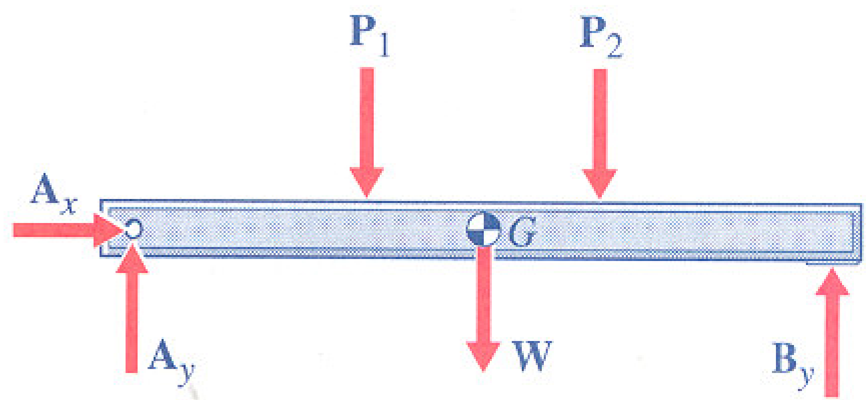


Figure (b)

- The beam is supported at the left end with a smooth pin and bracket and at the right end with a roller.
- The reaction of the left support is represented by the forces A_x and A_y.
- The reaction of the roller is represented by the force B_y, which acts normal to the surface of the beam.

Free-Body Diagram Example 3-2

Figure 3-2 shows a simplified rendition of a gear reducer where the input and output shafts AB and CD are rotating at constant speed ω_i and ω_o, respectively. The input and output torques (torsional moments) are T_i = 28 N.m and T_o, respectively. The shafts are supported in the housing by bearings at A, B, C and D. The pitch radii of gears G_1 and G_2 are r_1 = 20 mm and r_2 = 40 mm, respectively. **Draw the FBD of each member and determine the net reaction forces and moments at all points.**

Solution

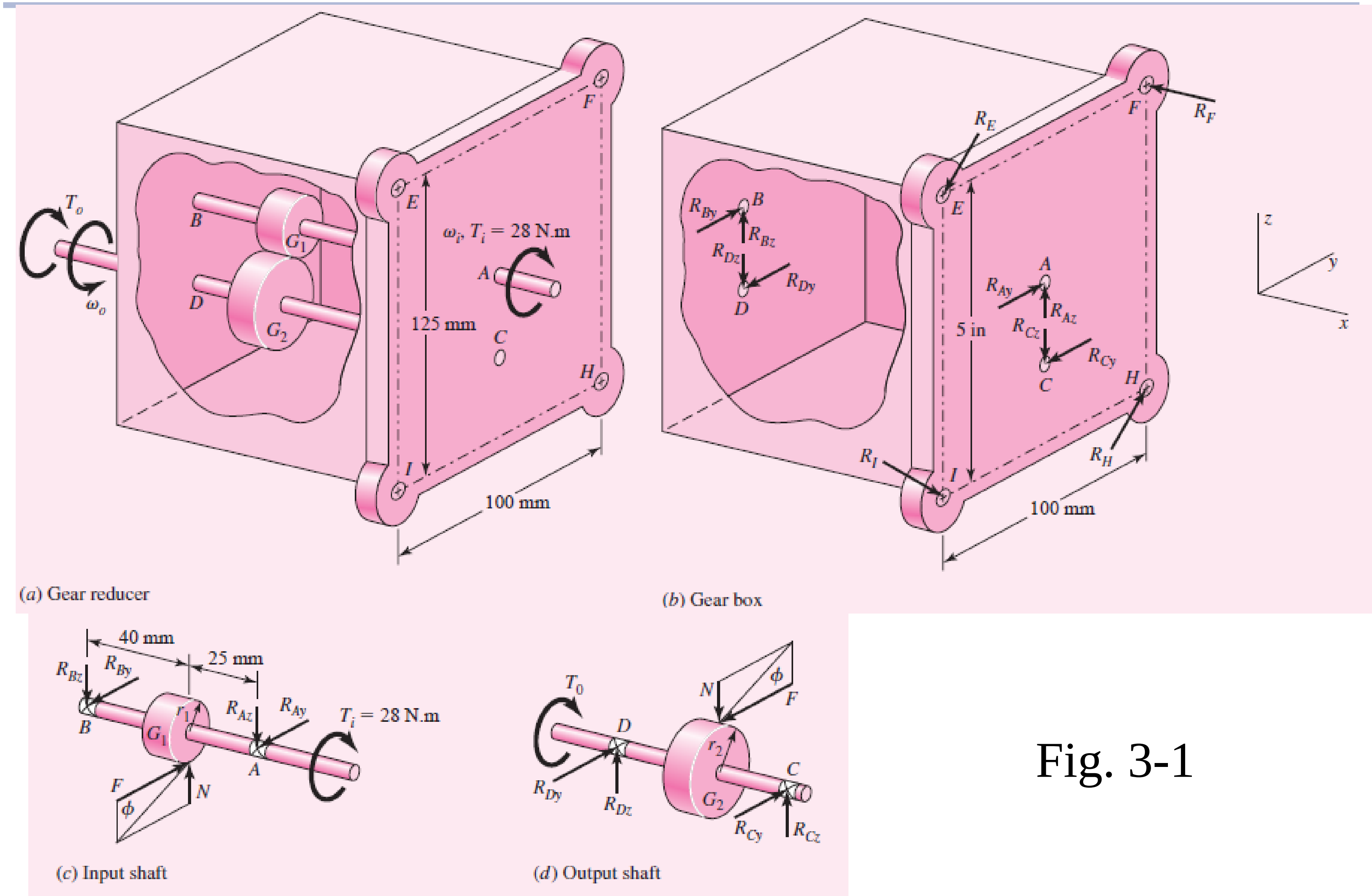
Simplifying assumptions:

1. Gears G_1 and G_2 are spur gears with a standard pressure angle $\phi = 20^o$.
2. The bearings are self aligning and the shafts can be considered to be simply supported.
3. The weight of each member is negligible.
4. Friction is negligible.
5. The mounting bolts at E, F, H and I are of the same size.

The separate FBD of the members are shown in Figs 4-4 b-d. The force transmitted between the spur gears is not tangential but at the pressure angle ϕ. Thus,

$$N = F \tan \phi$$

Free-Body Diagram Example 3-1



(a) Gear reducer

(b) Gear box

(c) Input shaft

(d) Output shaft

Fig. 3-1

Free-Body Diagram Example 3-2

Input Shaft AB

$$\sum M_x = 0 \Rightarrow F(0.02) - 28 = 0 \Rightarrow F = 1400 \text{ N} \tag{2}$$

and the normal force is

$$N = F \tan\phi = 1400 \tan 20 = 509.6 \text{ N} \tag{3}$$

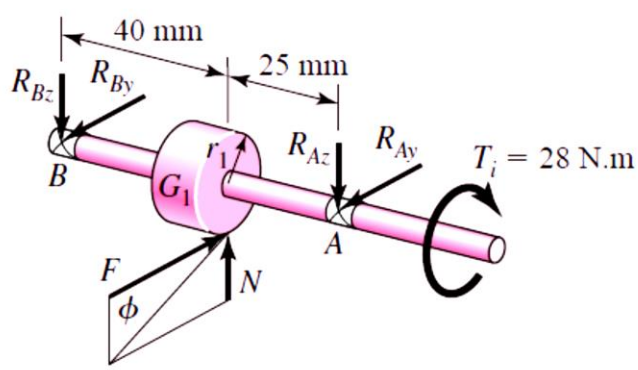
$$\sum F_y = 0 \Rightarrow R_{Ay} + R_{By} = F = 1400 \text{ N} \tag{4}$$

$$\sum F_z = 0 \Rightarrow R_{Az} + R_{Bz} = N = 509.6 \text{ N} \tag{5}$$

$$\sum M_z = 0 \Rightarrow R_{Ay}(65) - F(40) = 0 \Rightarrow R_{Ay} = 861.5 \text{ N} \tag{6}$$

$$\sum M_y = 0 \Rightarrow R_{Bz}(40) - R_{Az}(25) = 0 \Rightarrow R_{Az} = 1.6 R_{Bz} \tag{7}$$

Substitution of Eq. (6) into Eq. (4) gives $R_{By} = 538.5$ N. Similarly, substitution of Eq. (7) into Eq. (5) gives $R_{Bz} = 196$ N and $R_{Az} = 313.6$ N



(c) Input shaft

Free-Body Diagram Example 3-2

Output Shaft CD

Following the same procedure,

$$\sum F_y = 0 \Rightarrow R_{Dy} + R_{Cy} = F = 1400 \text{ N.m} \quad (8)$$

$$\sum F_z = 0 \Rightarrow R_{Dz} + R_{Cz} = N = 509.6 \text{ N.m} \quad (9)$$

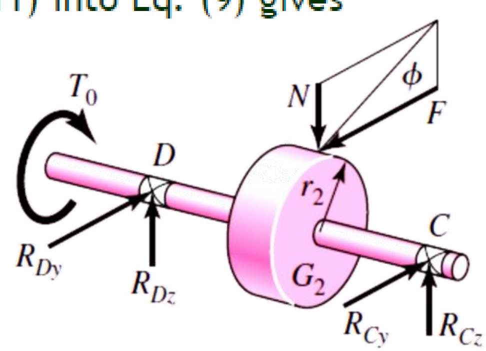
$$\sum M_z = 0 \Rightarrow R_{Cy}(65) - F(40) = 0 \Rightarrow R_{Cy} = 861.5 \text{ N} \quad (10)$$

$$\sum M_y = 0 \Rightarrow R_{Cz}(25) - R_{Dz}(40) = 0 \Rightarrow R_{Cz} = 1.6 R_{Dz} \quad (11)$$

therefore

$R_{Dy} = 1400\text{-}861.5 = 538.5 \text{ N}$. Similarly, substitution of Eq. (11) into Eq. (9) gives

$R_{Dz} = 196 \text{ N}$, therefore $R_{Cz} = 313.6 \text{ N}$.



(d) Output shaft

Free-Body Diagram Example 3-2

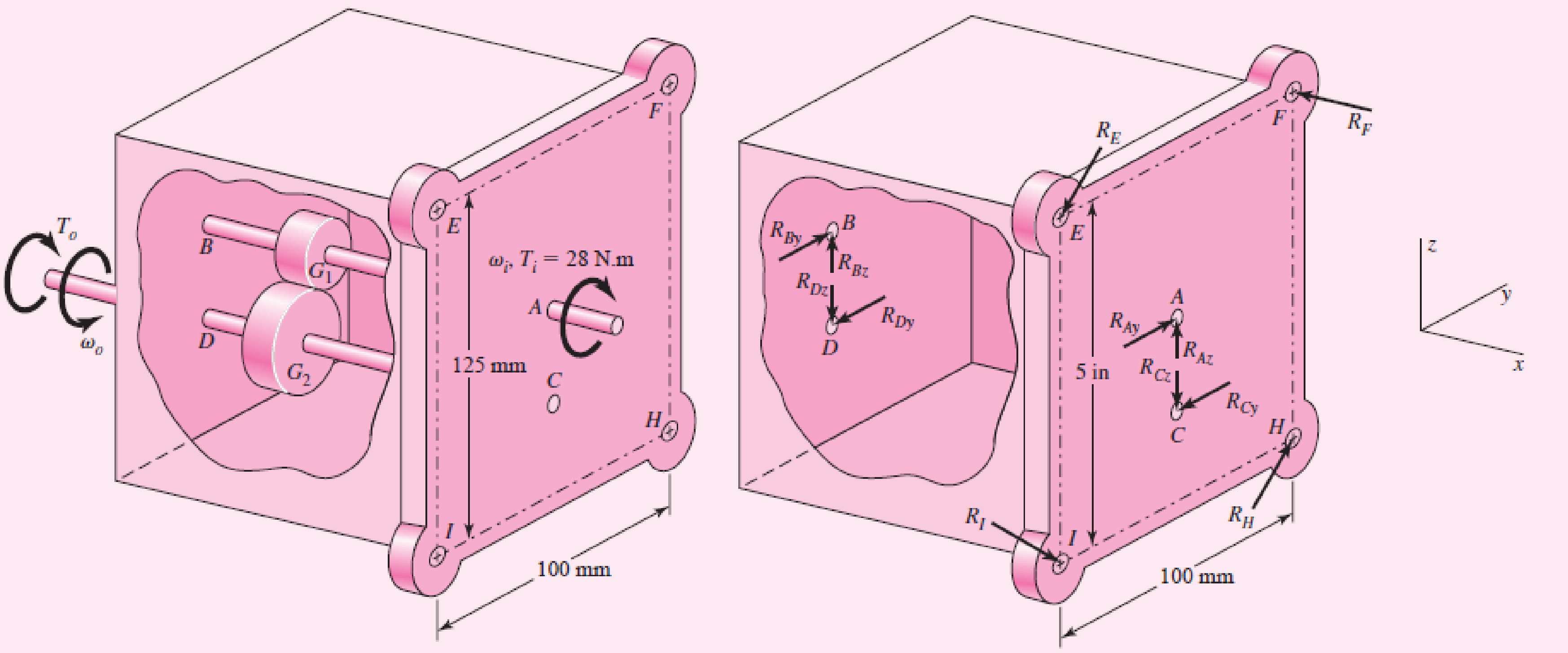
The output moment is

$$T_o = 28 + 1400\ (\ 0.02) = 56\ \text{N.m}$$

Notice that in Fig.4-4(b) the net force from bearing reactions is zero whereas the net moment about the x-axis is

$$T = (r_1 + r_2)R_{cy} + (r_1 + r_2)R_{Dy} = (r_1 + r_2)(R_{cy} + R_{Dy})$$

$$T = 0.06\ (861.5) + 0.06\ (538.5) = 84\ \text{N.m}$$



(a) Gear reducer

(b) Gear box

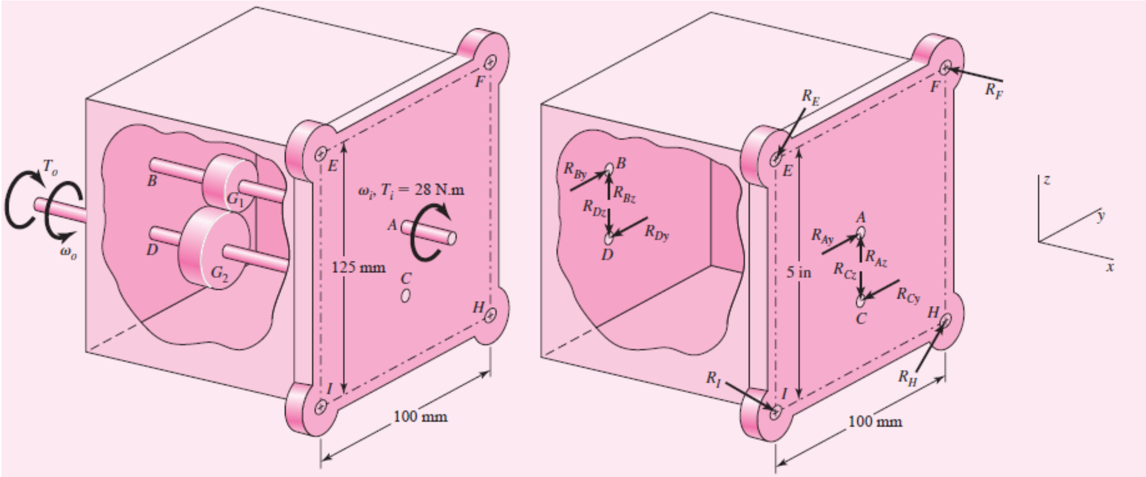
Gear Box

The reaction forces R_E, R_F, R_H, and R_I from the mounting bolts cannot be determined from the equilibrium equations as there are too many unknowns. Only three equations are available $\Sigma F_y = \Sigma F_z = \Sigma M_x = 0$. In case you were wondering about assumption 5, here is where we will use it. The gear box tends to rotate about the x-axis because of pure torsional moment of 84 N.m . The bolt forces must provide an equal but opposite torsional moment. The center of rotation relative to the bolts lies at the center of the centroid of the bolt cross-sectional area. Thus if the bolt areas are equal: the center of rotation is at the center of the four bolts, a distance of

$$\sqrt{\left(\frac{100}{2}\right)^2 + \left(\frac{125}{2}\right)^2} = 80\ mm$$

The bolt forces are equal ($R_E = R_F = R_H = R_I = R$), and each bolt force is perpendicular to the line from the bolt to the center of rotation. This gives a net torque from the four bolts of $4R\,(0.08)$ = 84 N.m .

Thus, $R_E = R_F = R_H = R_I =$ **262 N**



(a) Gear reducer

(b) Gear box

Shear Force and Bending Moments in Beams

Figure shows a beam supported by **reactions** R_1 and R_2 and loaded by the **concentrated forces** F_1 , F_2, and F_3. If the beam is cut at some section located at $x = x_1$ and the left hand portion is removed as a free body, an internal shear force V and bending moment M must act on the cut surface to ensure equilibrium.

- The shear force is obtained by summing the forces on the isolated sections.
- The bending moment is the sum of the moments of the forces to the left of the section taken about an axis through the isolated section.

The sign conventions used for bending moment and shear force are shown in Figure .

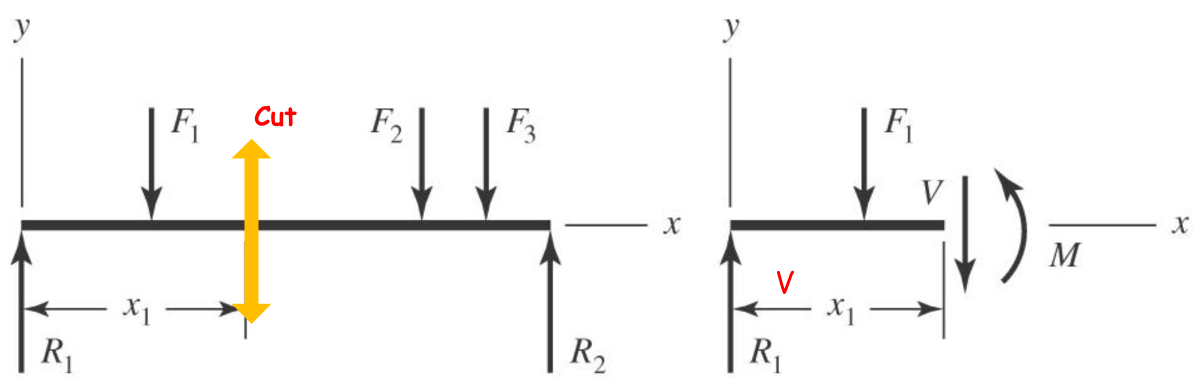
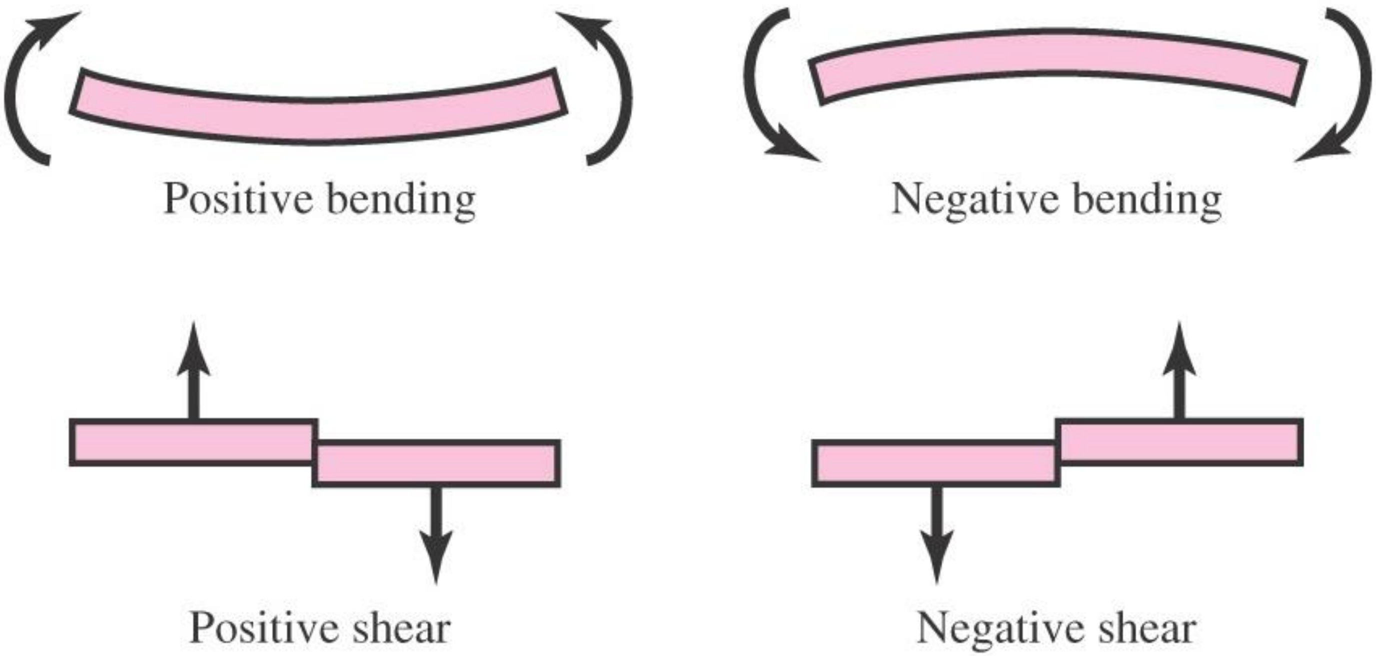


Figure FBD of simply supported beam with V and M shown in positive direction

Sign Conventions for Bending and Shear



Sign conventions for bending and shear.

Shear force and bending moment are related by

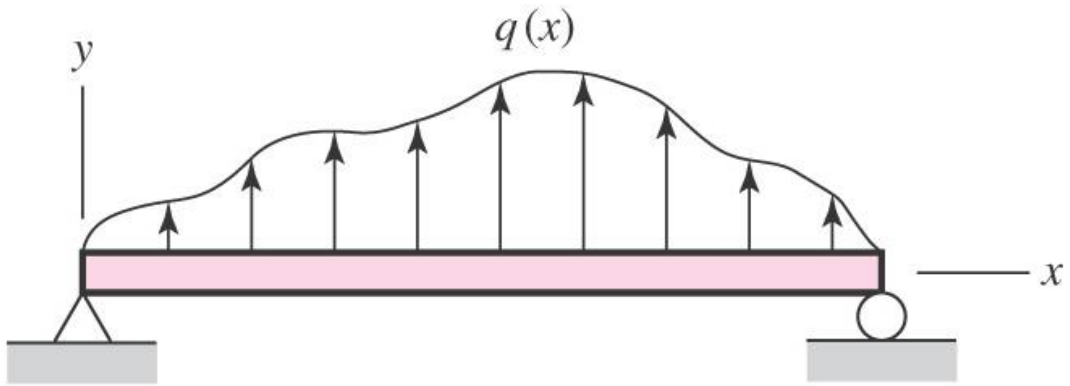
Shear Force

Bending moment

$$V = \frac{dM}{dx} \tag{3–3}$$

Distributed Load on Beam

- Sometimes the bending is caused by a distributed load ***q*(*x*)**
- Distributed load ***q*(*x*)** called *load intensity*
- Units of force per unit length and is positive in the positive y direction



Distributed load on beam.

$$\frac{dV}{dx} = \frac{d^2M}{dx^2} = q \qquad (3\text{–}4)$$

Relationships between Load, Shear, and Bending

Shear Force

Bending moment

$$V = \frac{dM}{dx} \tag{3–3}$$

$$\frac{dV}{dx} = \frac{d^2M}{dx^2} = q \tag{3–4}$$

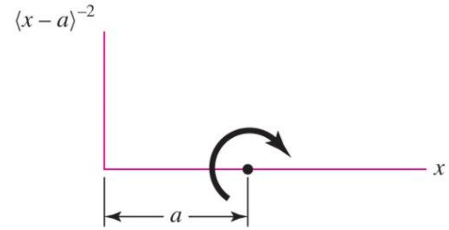
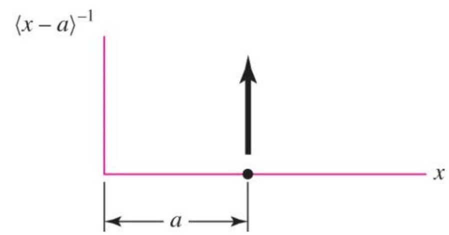
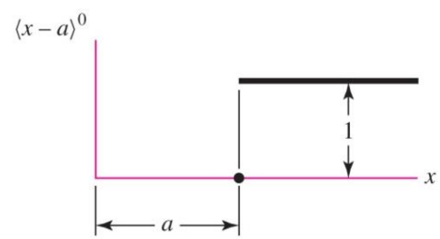
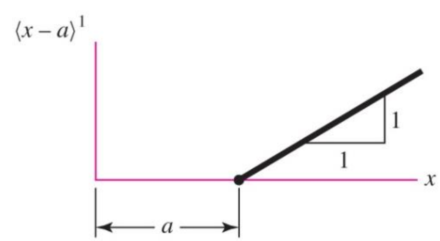
$$\int_{V_A}^{V_B} dV = V_B - V_A = \int_{x_A}^{x_B} q\,dx \tag{3–5}$$

$$\int_{M_A}^{M_B} dM = M_B - M_A = \int_{x_A}^{x_B} V\,dx \tag{3–6}$$

- The change in shear force from *A* to *B* is equal to the area of the loading diagram between x_A and x_B.
- The change in moment from *A* to *B* is equal to the area of the shear-force diagram between x_A and x_B.

Singularity Functions

- A notation useful for integrating across discontinuities
- Angle brackets indicate special function to determine whether forces and moments are active

Function	Graph of $f_n(x)$	Meaning
Concentrated moment (unit doublet)		$\langle x-a\rangle^{-2}=0 \quad x\neq a$ $\langle x-a\rangle^{-2}=\pm\infty \quad x=a$ $\int \langle x-a\rangle^{-2}\,dx=\langle x-a\rangle^{-1}$
Concentrated force (unit impulse)		$\langle x-a\rangle^{-1}=0 \quad x\neq a$ $\langle x-a\rangle^{-1}=+\infty \quad x=a$ $\int \langle x-a\rangle^{-1}\,dx=\langle x-a\rangle^{0}$
Unit step		$\langle x-a\rangle^{0}=\begin{cases}0 & x<a\\ 1 & x\geq a\end{cases}$ $\int \langle x-a\rangle^{0}\,dx=\langle x-a\rangle^{1}$
Ramp		$\langle x-a\rangle^{1}=\begin{cases}0 & x<a\\ x-a & x\geq a\end{cases}$ $\int \langle x-a\rangle^{1}\,dx=\frac{\langle x-a\rangle^{2}}{2}$

†W. H. Macaulay, "Note on the deflection of beams," *Messenger of Mathematics,* vol. 48, pp. 129–130, 1919.

Table 3−1

Shear-Moment Diagrams

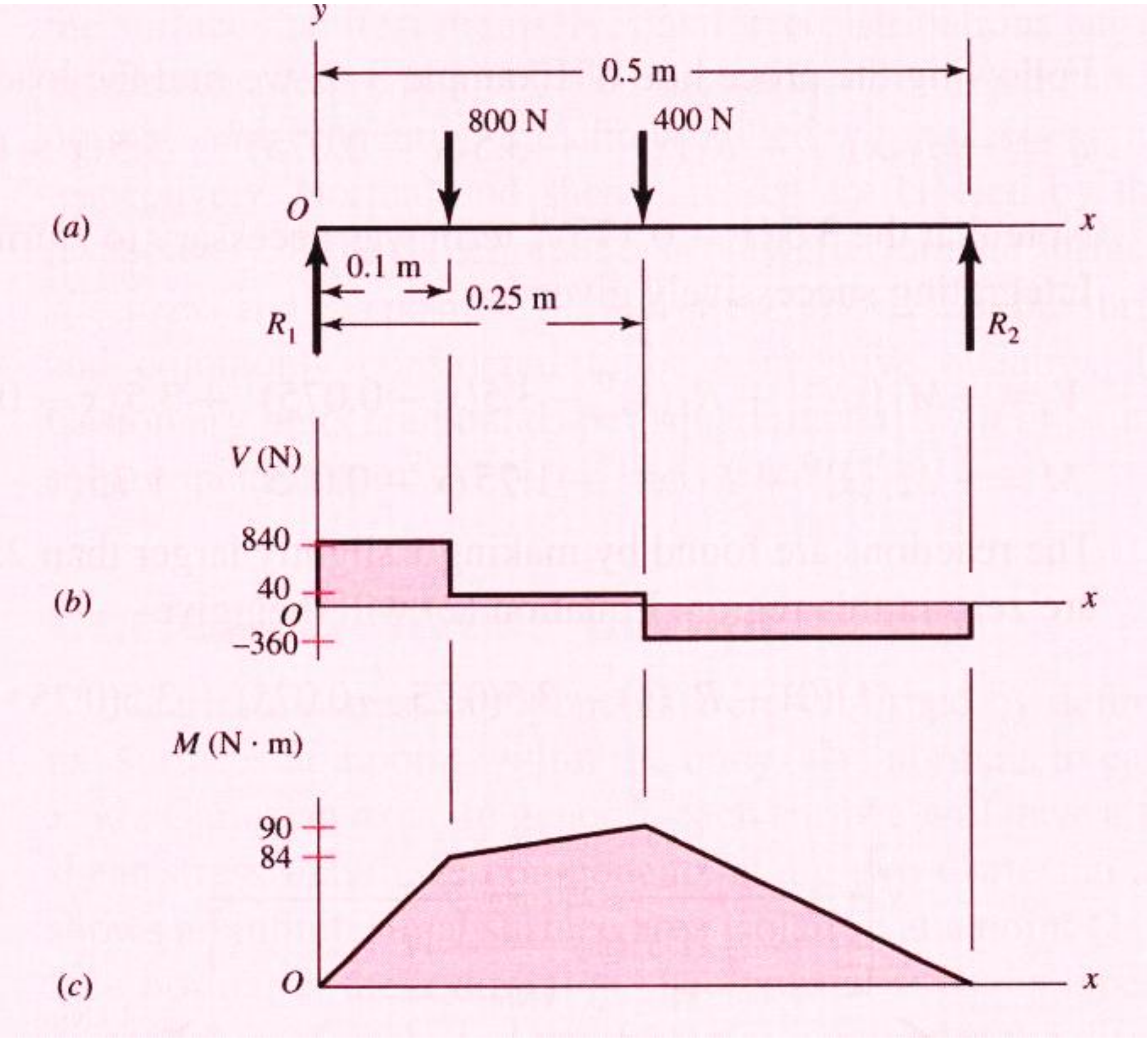


Fig. 3−5

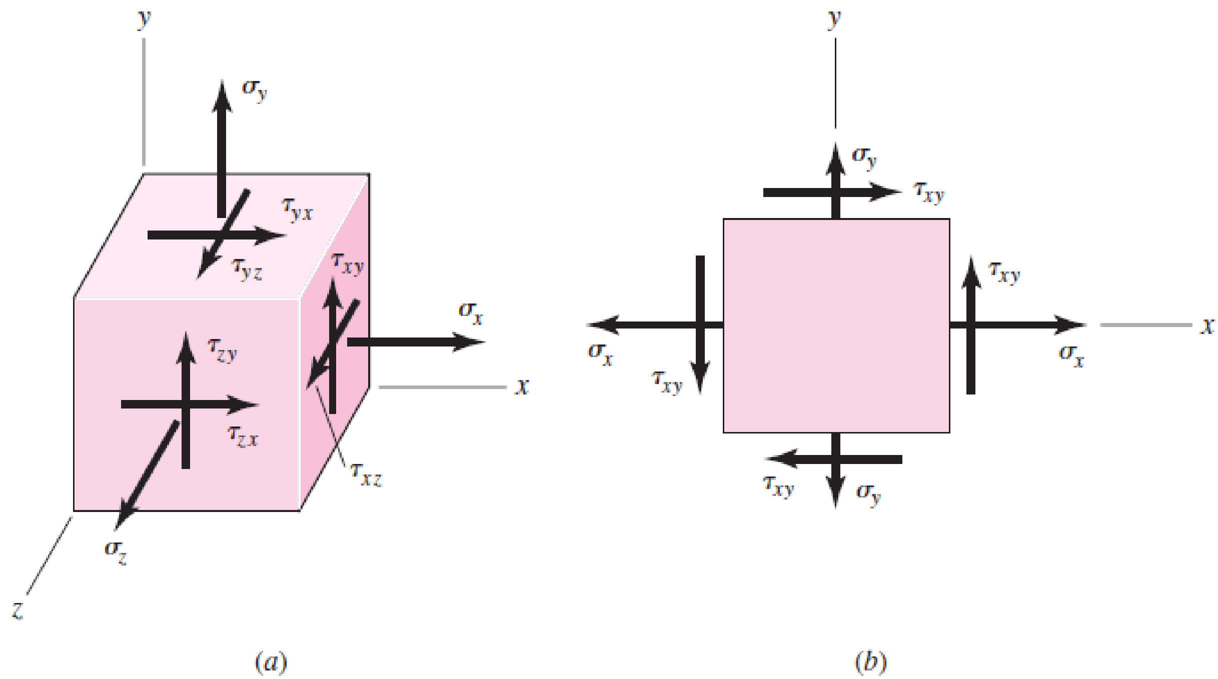
Stress

- *Normal stress* is normal to a surface, designated by σ
- *Tangential shear stress* is tangent to a surface, designated by τ
- Normal stress acting outward on surface is *tensile stress*
- Normal stress acting inward on surface is *compressive stress*
- U.S. Customary units of stress are pounds per square inch (psi)
- SI units of stress are newtons per square meter (N/m^2)
- 1 N/m^2 = 1 pascal (Pa)

Stress element

Figure 3–8

(*a*) General three-dimensional stress. (*b*) Plane stress with "cross-shears" equal.



- Represents stress *at a point*
- Coordinate directions are arbitrary
- Choosing coordinates which result in zero shear stress will produce principal stresses

Cartesian Stress Components

- Defined by three mutually orthogonal surfaces at a point within a body
- Each surface can have normal and shear stress
- Shear stress is often resolved into perpendicular components
- First subscript indicates direction of surface normal
- Second subscript indicates direction of shear stress

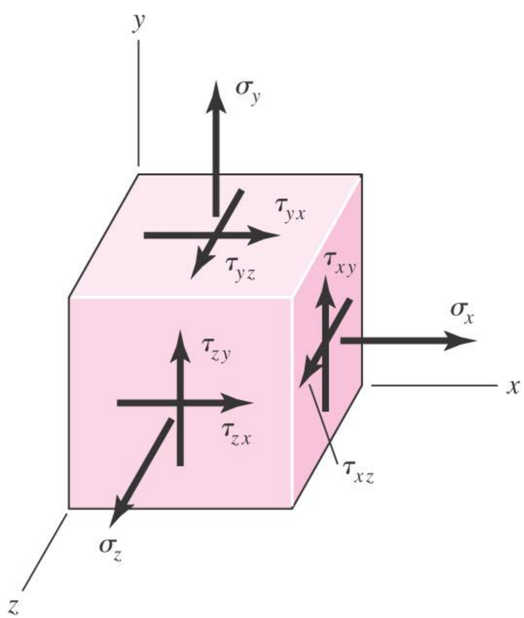


Fig. 3−8 (a)

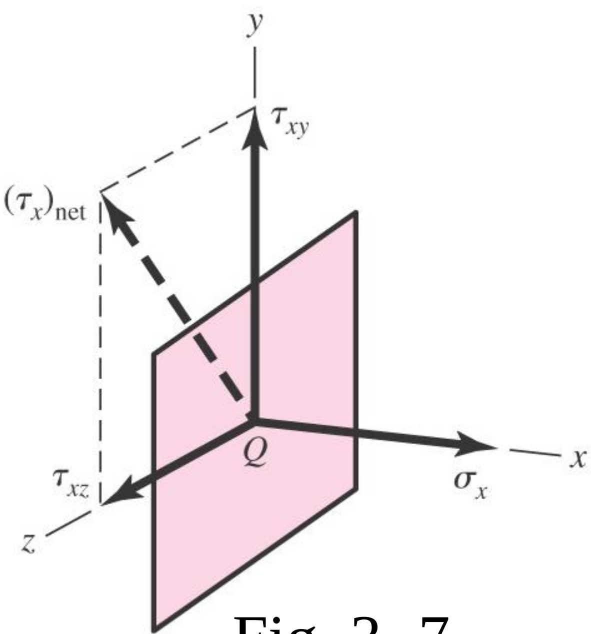
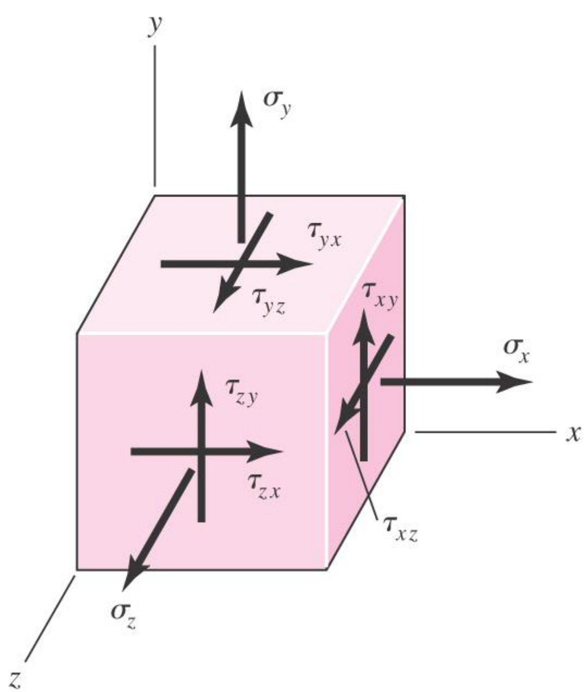
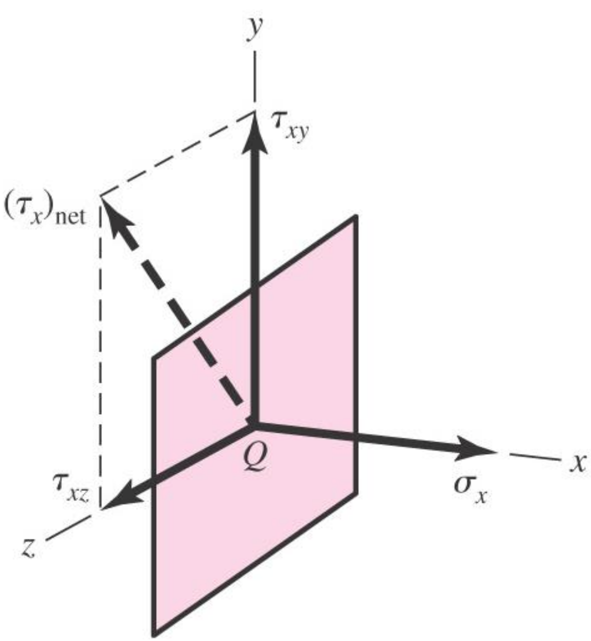


Fig. 3−7

Cartesian Stress Components

- Defined by three mutually orthogonal surfaces at a point within a body
- Each surface can have normal and shear stress
- Shear stress is often resolved into perpendicular components
- First subscript indicates direction of surface normal
- Second subscript indicates direction of shear stress





Cartesian Stress Components

- In most cases, "cross shears" are equal

$$\tau_{yx} = \tau_{xy} \qquad \tau_{zy} = \tau_{yz} \qquad \tau_{xz} = \tau_{zx} \tag{3–7}$$

- This reduce the number of stress components from nine to six quantities, σ_x, σ_y, σ_z, τ_{xy}, τ_{yz}, and τ_{zx}.
- *Plane stress* occurs when stresses on one surface are zero

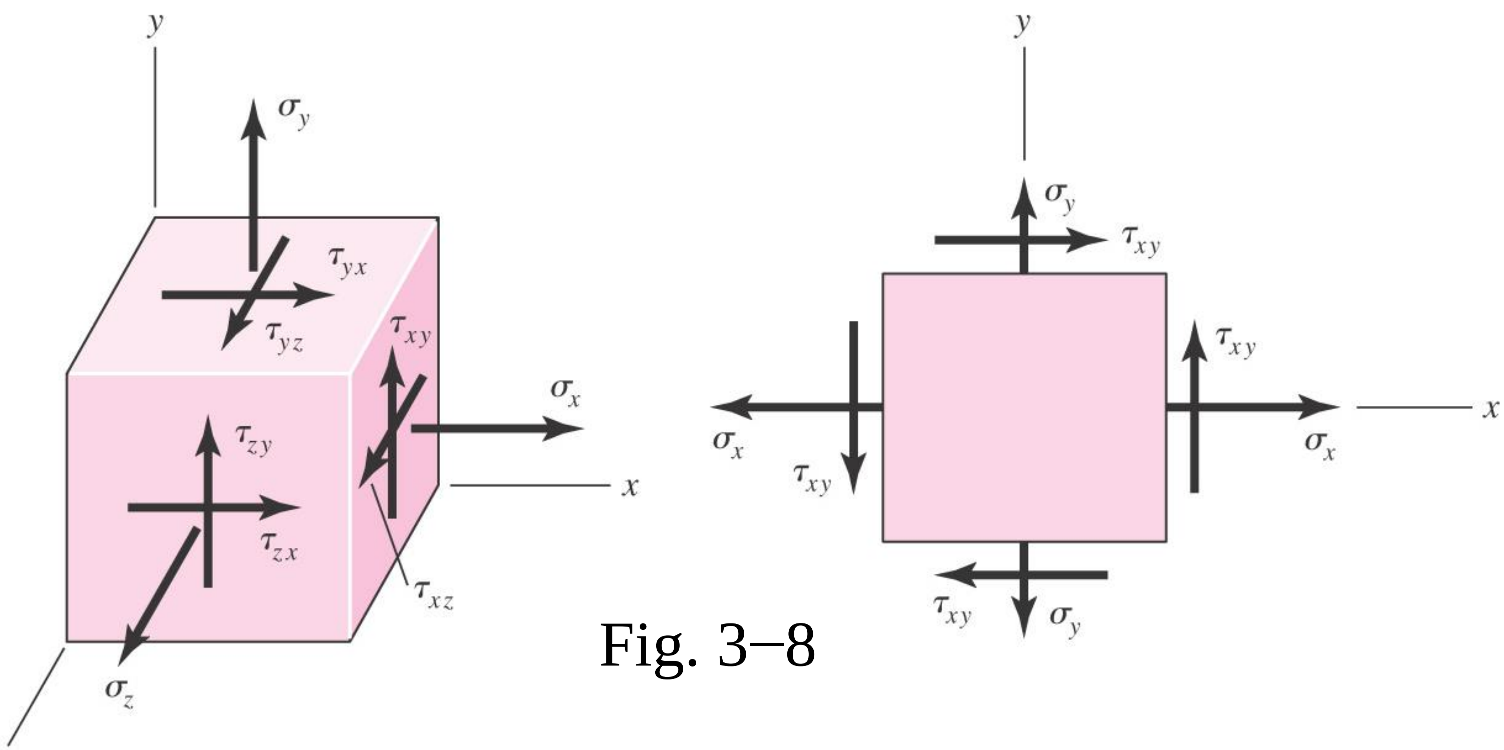


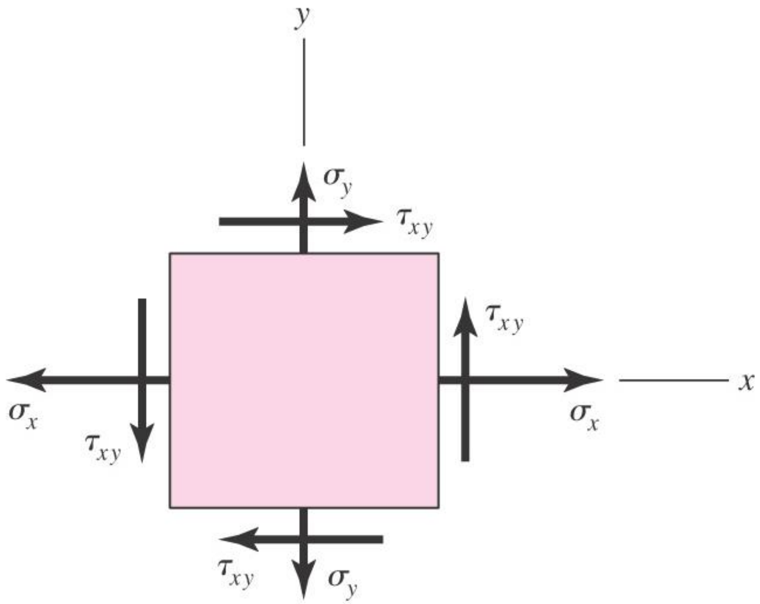
Fig. 3–8

Plane-Stress Transformation Equations

- Cutting plane stress element at an arbitrary angle and balancing stresses gives *plane-stress transformation equations*

$$\sigma = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\phi + \tau_{xy}\sin 2\phi \qquad (3\text{–}8)$$

$$\tau = -\frac{\sigma_x - \sigma_y}{2}\sin 2\phi + \tau_{xy}\cos 2\phi \qquad (3\text{–}9)$$



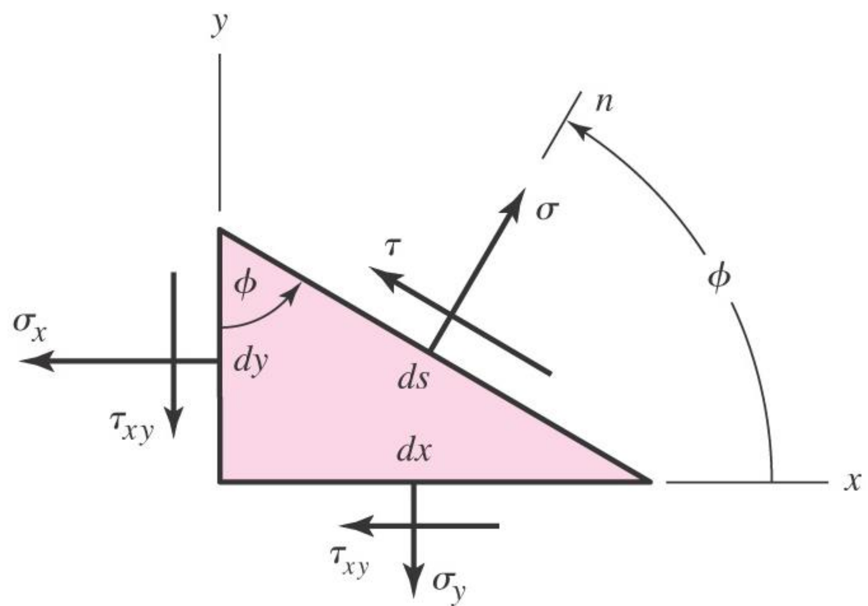


Fig. 3–9

Principal Stresses for Plane Stress

- Differentiating Eq. (3-8) with respect to ϕ and setting equal to zero maximizes σ and gives

$$\tan 2\phi_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \tag{3–10}$$

- The two values of $2\phi_p$ are the *principal directions.*
- The stresses in the principal directions are the *principal stresses*.
- The principal direction surfaces have zero shear stresses.
- Substituting Eq. (3-10) into Eq. (3-8) gives expression for the non-zero principal stresses.

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \tag{3–13}$$

- Note that there is a third principal stress, equal to zero for plane stress.

Extreme-value Shear Stresses for Plane Stress

- Performing similar procedure with shear stress in Eq. (3-9), the maximum shear stresses are found to be on surfaces that are ±45° from the principal directions.
- The two extreme-value shear stresses are

$$\tau_1, \tau_2 = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad (3\text{–}14)$$

Mohr's Circle Diagram

- A graphical method for visualizing the stress state at a point
- Represents relation between x-y stresses and principal stresses
- Parametric relationship between σ and τ (with 2ϕ as parameter)
- Relationship is a circle with center at

$$C = (\sigma, \tau) = [(\sigma_x + \sigma_y)/2, 0]$$

and radius of

$$R = \sqrt{\left[\frac{\sigma_x - \sigma_y}{2}\right]^2 + \tau_{xy}^2}$$

Mohr's Circle Diagram

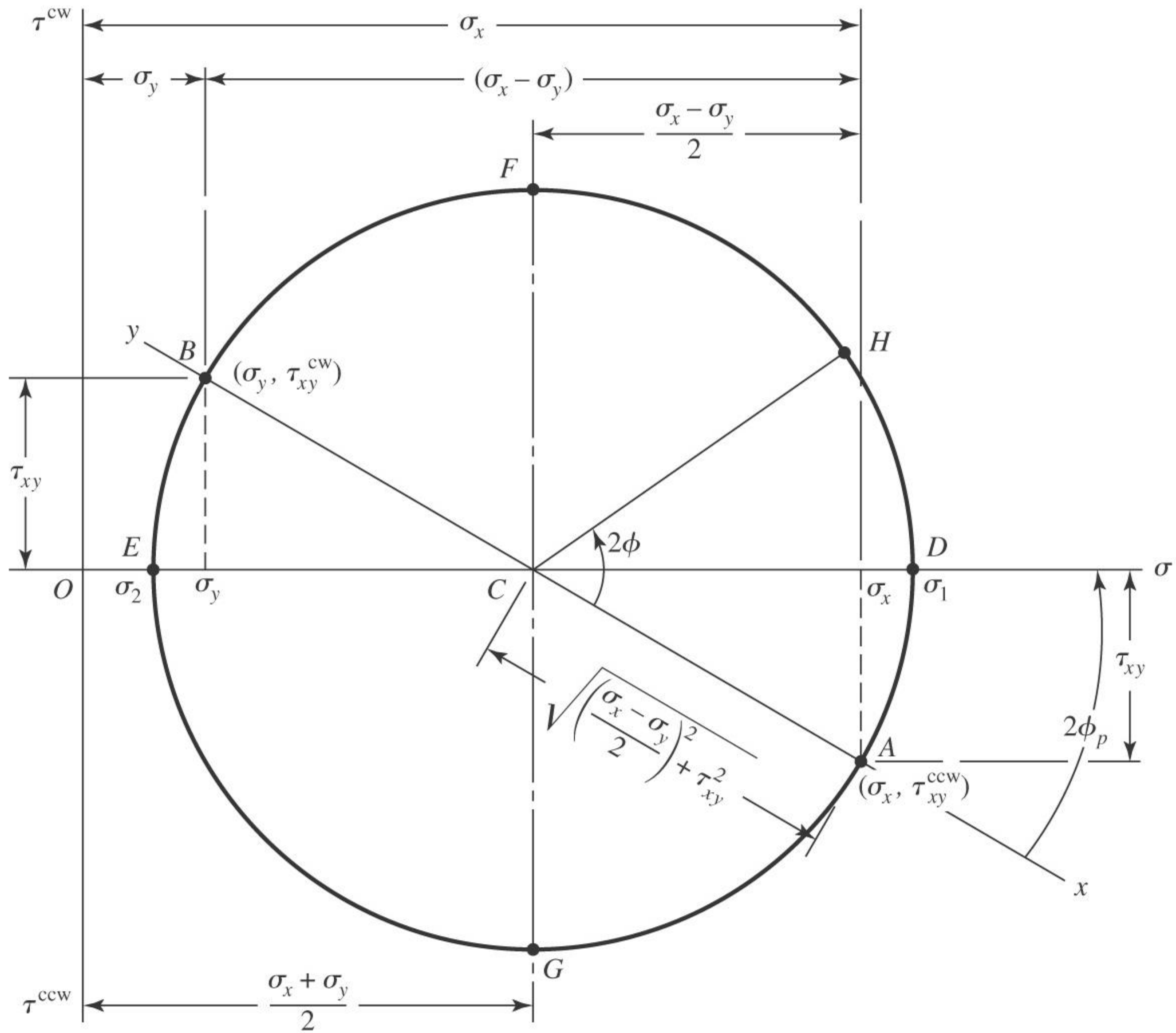


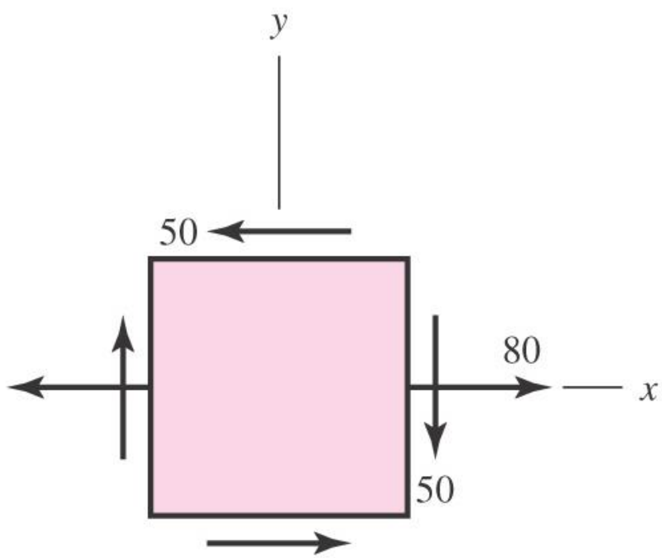
Fig. 3−10

Example 3-4

A stress element has $\sigma_x = 80$ MPa and $\tau_{xy} = 50$ MPa cw, as shown in Fig. 3–11*a*.

(*a*) Using Mohr's circle, find the principal stresses and directions, and show these on a stress element correctly aligned with respect to the xy coordinates. Draw another stress element to show τ_1 and τ_2, find the corresponding normal stresses, and label the drawing completely.

(*b*) Repeat part *a* using the transformation equations only.



(*a*)

Fig. 3−11

Example 3-4

(a) In the semigraphical approach used here, we first make an approximate freehand sketch of Mohr's circle and then use the geometry of the figure to obtain the desired information.

Draw the σ and τ axes first (Fig. 3–11b) and from the x face locate $\sigma_x = 80$ MPa along the σ axis. On the x face of the element, we see that the shear stress is 50 MPa in the cw direction. Thus, for the x face, this establishes point A (80, 50^{cw}) MPa. Corresponding to the y face, the stress is $\sigma = 0$ and $\tau = 50$ MPa in the ccw direction. This locates point B (0, 50^{ccw}) MPa. The line AB forms the diameter of the required circle, which can now be drawn. The intersection of the circle with the σ axis defines σ_1 and σ_2 as shown. Now, noting the triangle ACD, indicate on the sketch the length of the legs AD and CD as 50 and 40 MPa, respectively. The length of the hypotenuse AC is

$$\tau_1 = \sqrt{(50)^2 + (40)^2} = 64.0 \text{ MPa}$$

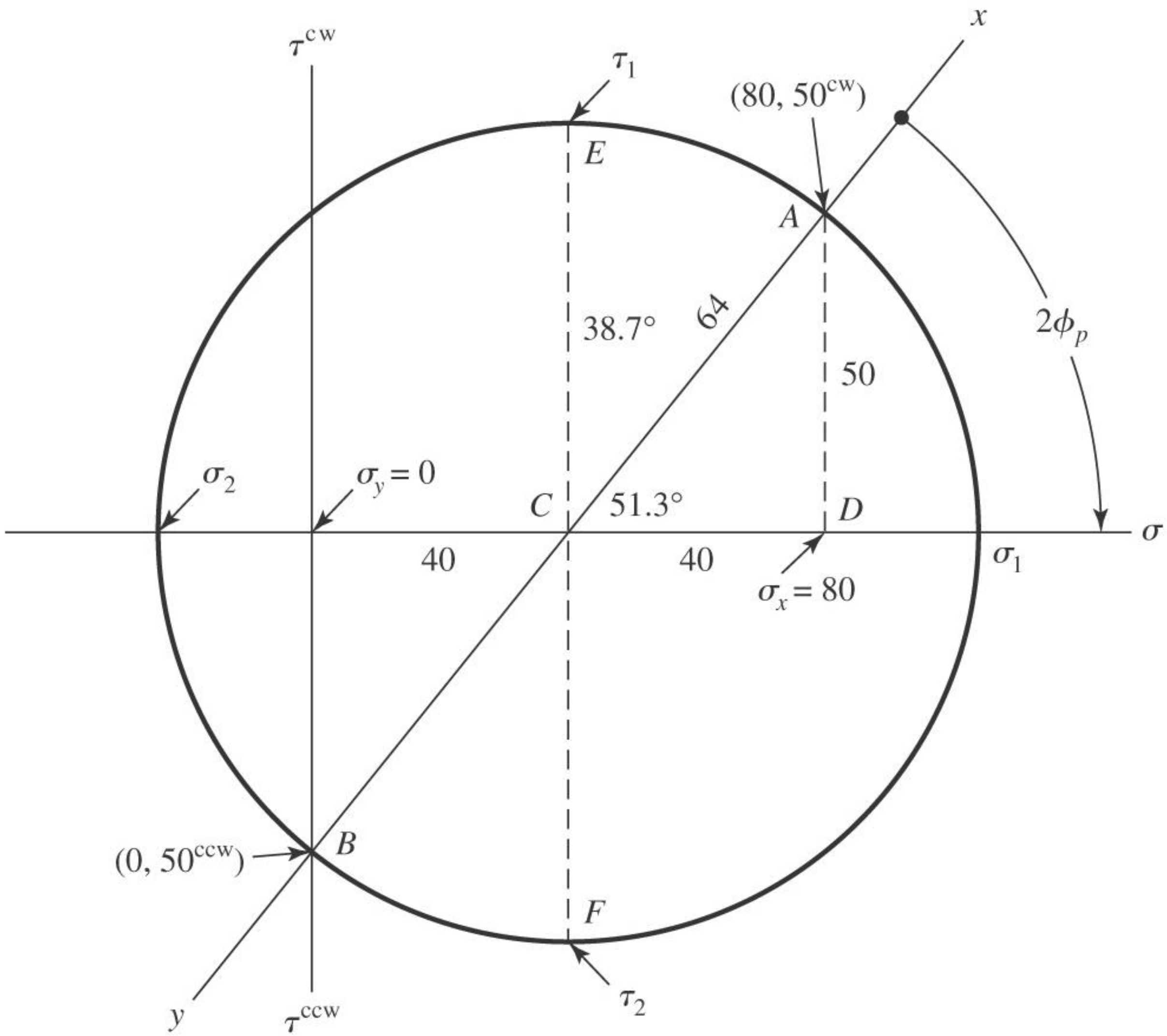
and this should be labeled on the sketch too. Since intersection C is 40 MPa from the origin, the principal stresses are now found to be

$$\sigma_1 = 40 + 64 = 104 \text{ MPa} \qquad \text{and} \qquad \sigma_2 = 40 - 64 = -24 \text{ MPa}$$

The angle 2ϕ from the x axis cw to σ_1 is

$$2\phi_p = \tan^{-1} \tfrac{50}{40} = 51.3^\circ$$

Example 3-4

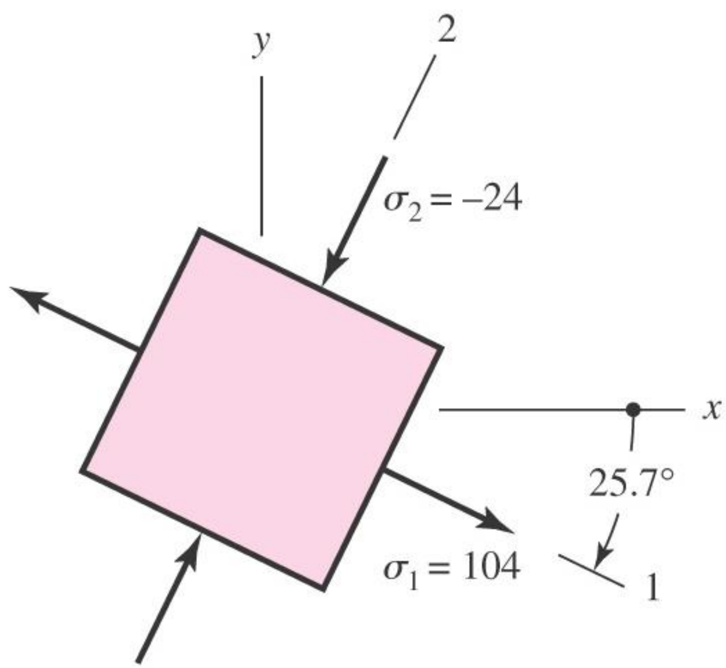


(b)

Fig. 3−11

Example 3-4

To draw the principal stress element (Fig. 3–11*c*), sketch the x and y axes parallel to the original axes. The angle ϕ_p on the stress element must be measured in the *same* direction as is the angle $2\phi_p$ on the Mohr circle. Thus, from x measure 25.7° (half of 51.3°) clockwise to locate the σ_1 axis. The σ_2 axis is 90° from the σ_1 axis and the stress element can now be completed and labeled as shown. Note that there are *no* shear stresses on this element.



(*c*)

Fig. 3−11

Example 3-4

The two maximum shear stresses occur at points E and F in Fig. 3–11b. The two normal stresses corresponding to these shear stresses are each 40 MPa, as indicated. Point E is 38.7° ccw from point A on Mohr's circle. Therefore, in Fig. 3–11d, draw a stress element oriented 19.3° (half of 38.7°) ccw from x. The element should then be labeled with magnitudes and directions as shown.

In constructing these stress elements it is important to indicate the x and y directions of the original reference system. This completes the link between the original machine element and the orientation of its principal stresses.

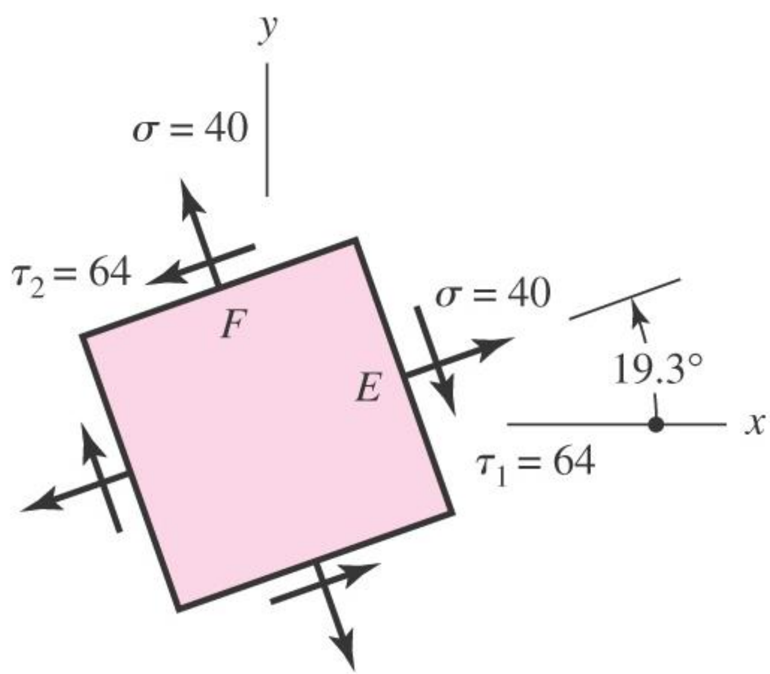


Fig. 3−11(d)

Example 3-4

(*b*) The transformation equations are programmable. From Eq. (3–10),

$$\phi_p = \frac{1}{2}\tan^{-1}\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right) = \frac{1}{2}\tan^{-1}\left(\frac{2(-50)}{80}\right) = -25.7^\circ, 64.3^\circ$$

From Eq. (3–8), for the first angle $\phi_p = -25.7^\circ$,

$$\sigma = \frac{80+0}{2} + \frac{80-0}{2}\cos[2(-25.7)] + (-50)\sin[2(-25.7)] = 104.03\ \text{MPa}$$

The shear on this surface is obtained from Eq. (3–9) as

$$\tau = -\frac{80-0}{2}\sin[2(-25.7)] + (-50)\cos[2(-25.7)] = 0\ \text{MPa}$$

which confirms that 104.03 MPa is a principal stress. From Eq. (3–8), for $\phi_p = 64.3^\circ$,

$$\sigma = \frac{80+0}{2} + \frac{80-0}{2}\cos[2(64.3)] + (-50)\sin[2(64.3)] = -24.03\ \text{MPa}$$

Example 3-4

Substituting $\phi_p = 64.3^\circ$ into Eq. (3–9) again yields $\tau = 0$, indicating that -24.03 MPa is also a principal stress. Once the principal stresses are calculated they can be ordered such that $\sigma_1 \geq \sigma_2$. Thus, $\sigma_1 = 104.03$ MPa and $\sigma_2 = -24.03$ MPa.

Since for $\sigma_1 = 104.03$ MPa, $\phi_p = -25.7^\circ$, and since ϕ is defined positive ccw in the transformation equations, we rotate *clockwise* 25.7° for the surface containing σ_1. We see in Fig. 3–11*c* that this totally agrees with the semigraphical method.

To determine τ_1 and τ_2, we first use Eq. (3–11) to calculate ϕ_s:

$$\phi_s = \frac{1}{2}\tan^{-1}\left(-\frac{\sigma_x - \sigma_y}{2\tau_{xy}}\right) = \frac{1}{2}\tan^{-1}\left(-\frac{80}{2(-50)}\right) = 19.3^\circ, 109.3^\circ$$

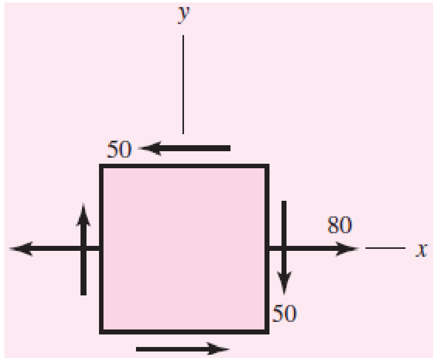
For $\phi_s = 19.3^\circ$, Eqs. (3–8) and (3–9) yield

$$\sigma = \frac{80+0}{2} + \frac{80-0}{2}\cos[2(19.3)] + (-50)\sin[2(19.3)] = 40.0 \text{ MPa}$$

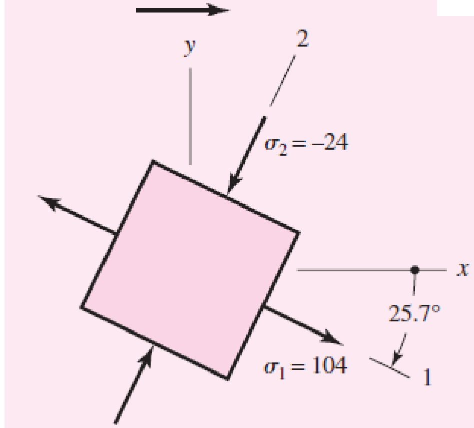
$$\tau = -\frac{80-0}{2}\sin[2(19.3)] + (-50)\cos[2(19.3)] = -64.0 \text{ MPa}$$

Example 3-4 Summary

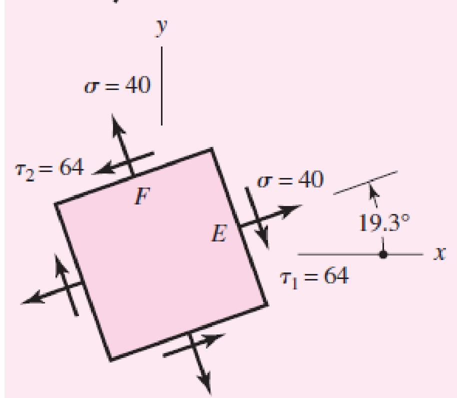
x-*y* orientation

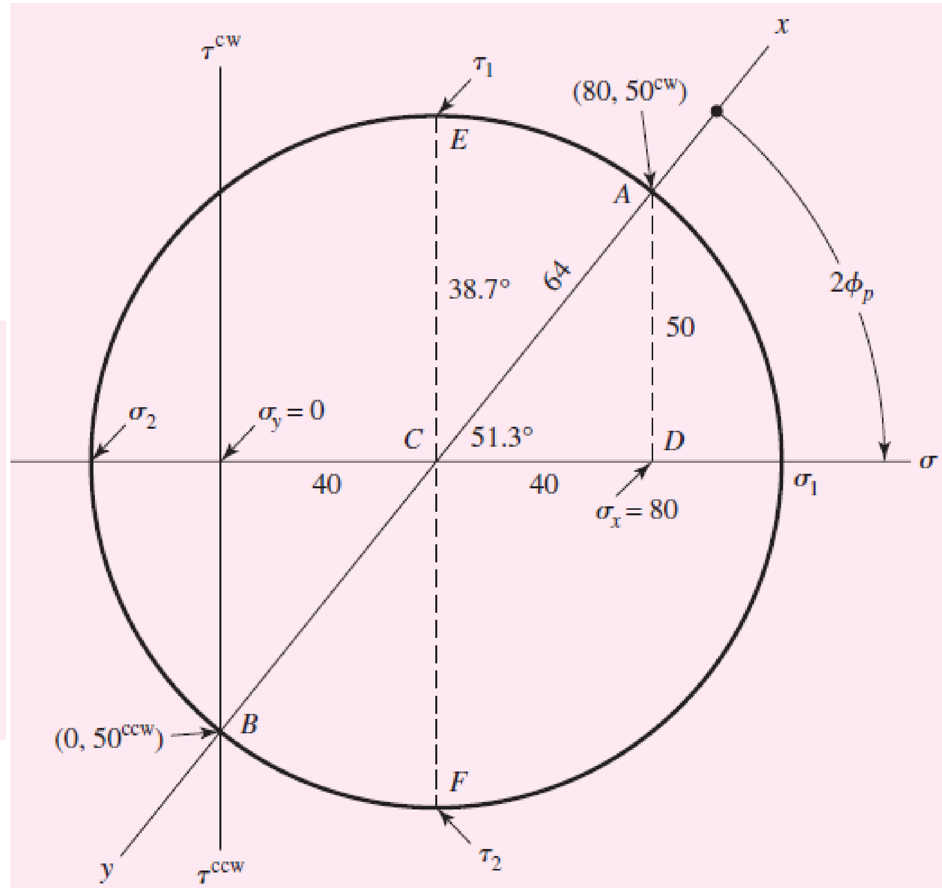


Principal stress orientation



Max shear orientation





Problem 1

For each of the plane stress states listed below, draw a Mohr's circle diagram properly labeled, find the principal normal and shear stresses, and determine the angle from the x axis to σ_1. Draw stress elements as in Fig. 3–11c and d and label all details.

(a) $\sigma_x = 20\,\text{kpsi}$, $\sigma_y = -10\,\text{kpsi}$, $\tau_{xy} = 8\,\text{kpsi cw}$

(b) $\sigma_x = 16\,\text{kpsi}$, $\sigma_y = 9\,\text{kpsi}$, $\tau_{xy} = 5\,\text{kpsi ccw}$

(c) $\sigma_x = 10\,\text{kpsi}$, $\sigma_y = 24\,\text{kpsi}$, $\tau_{xy} = 6\,\text{kpsi ccw}$

(d) $\sigma_x = -12\,\text{kpsi}$, $\sigma_y = 22\,\text{kpsi}$, $\tau_{xy} = 12\,\text{kpsi cw}$

(a)

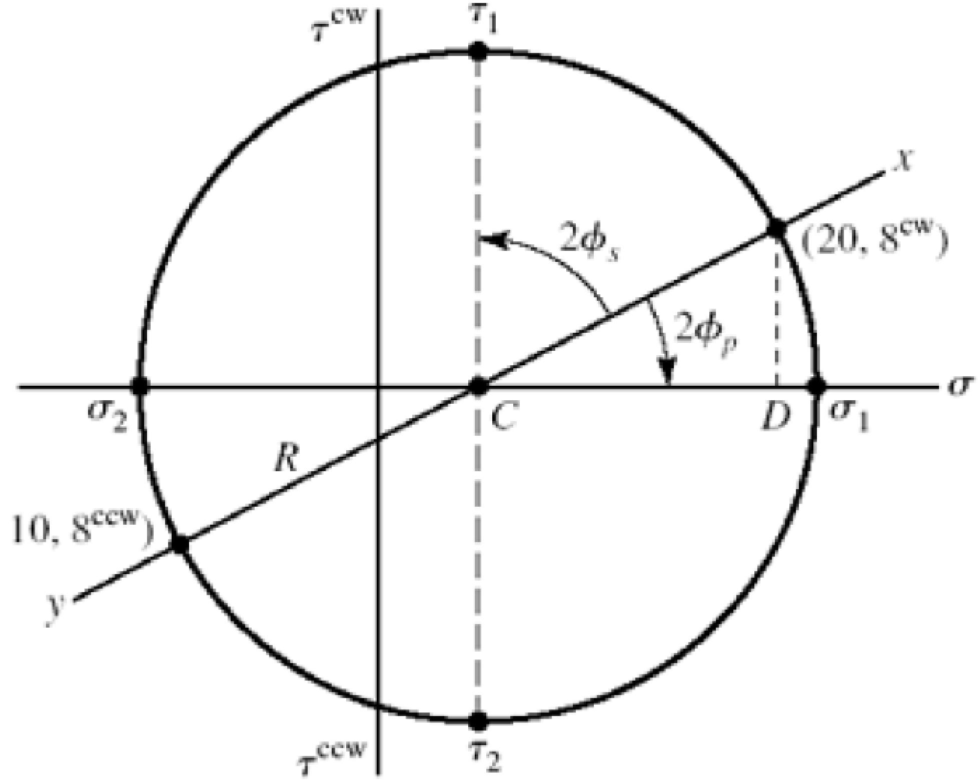
$$C = \frac{20-10}{2} = 5 \text{ kpsi}$$

$$CD = \frac{20+10}{2} = 15 \text{ kpsi}$$

$$R = \sqrt{15^2 + 8^2} = 17 \text{ kpsi}$$

$$\sigma_1 = 5 + 17 = 22 \text{ kpsi}$$

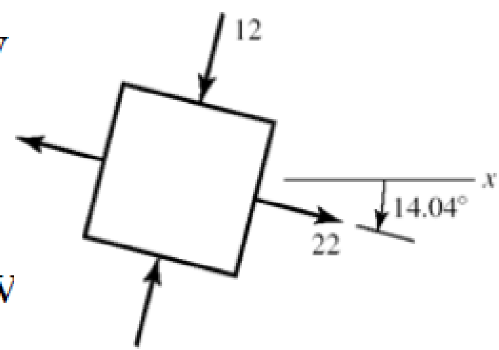
$$\sigma_2 = 5 - 17 = -12 \text{ kpsi}$$

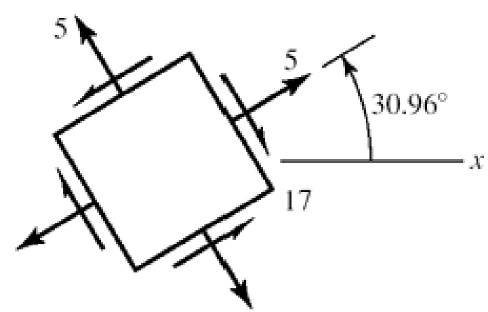


$$\phi_p = \frac{1}{2}\tan^{-1}\left(\frac{8}{15}\right) = 14.04^\circ \text{ cw}$$

$$\tau_1 = R = 17 \text{ kpsi}$$

$$\phi_s = 45^\circ - 14.04^\circ = 30.96^\circ \text{ ccw}$$





(b)

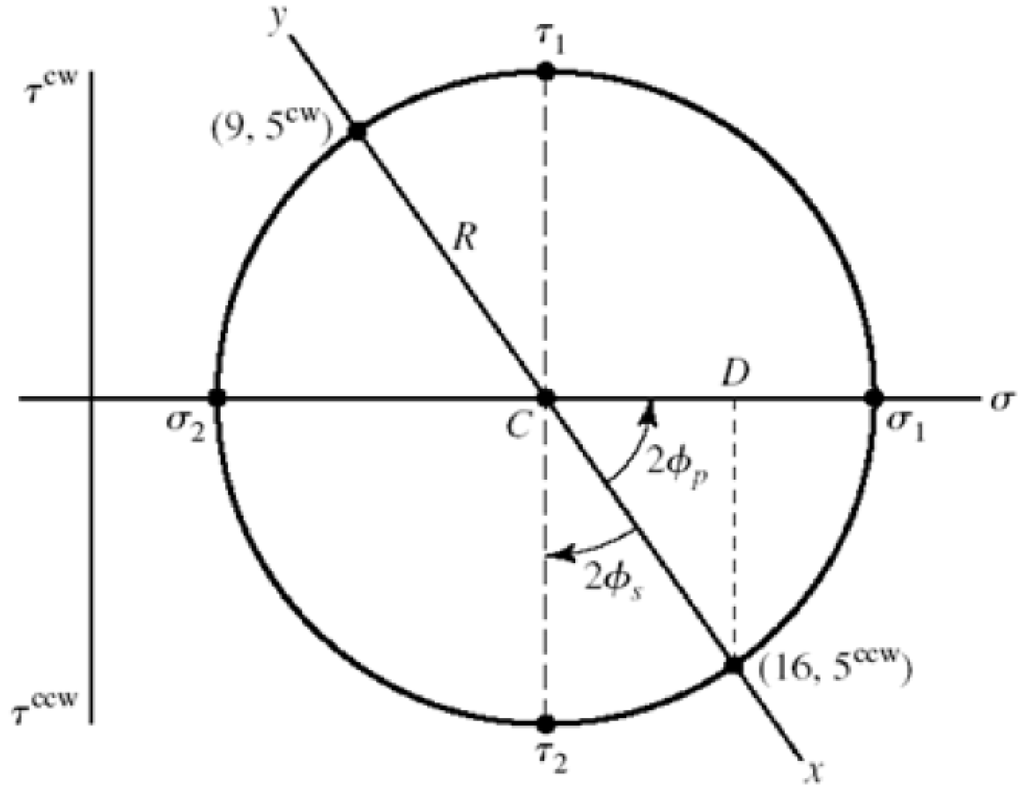
$$C = \frac{9+16}{2} = 12.5 \text{ kpsi}$$

$$CD = \frac{16-9}{2} = 3.5 \text{ kpsi}$$

$$R = \sqrt{5^2 + 3.5^2} = 6.10 \text{ kpsi}$$

$$\sigma_1 = 12.5 + 6.1 = 18.6 \text{ kpsi}$$

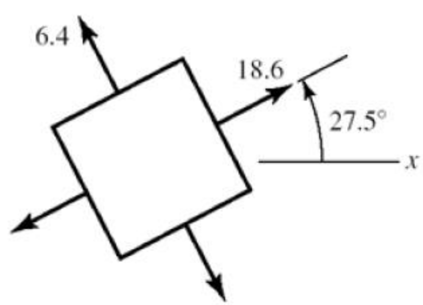
$$\sigma_2 = 12.5 - 6.1 = 6.4 \text{ kpsi}$$

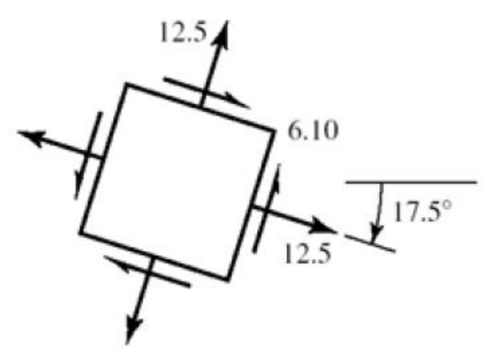


$$\phi_p = \frac{1}{2}\tan^{-1}\left(\frac{5}{3.5}\right) = 27.5^\circ \text{ ccw}$$

$$\tau_1 = R = 6.10 \text{ kpsi}$$

$$\phi_s = 45^\circ - 27.5^\circ = 17.5^\circ \text{ cw}$$





(c)

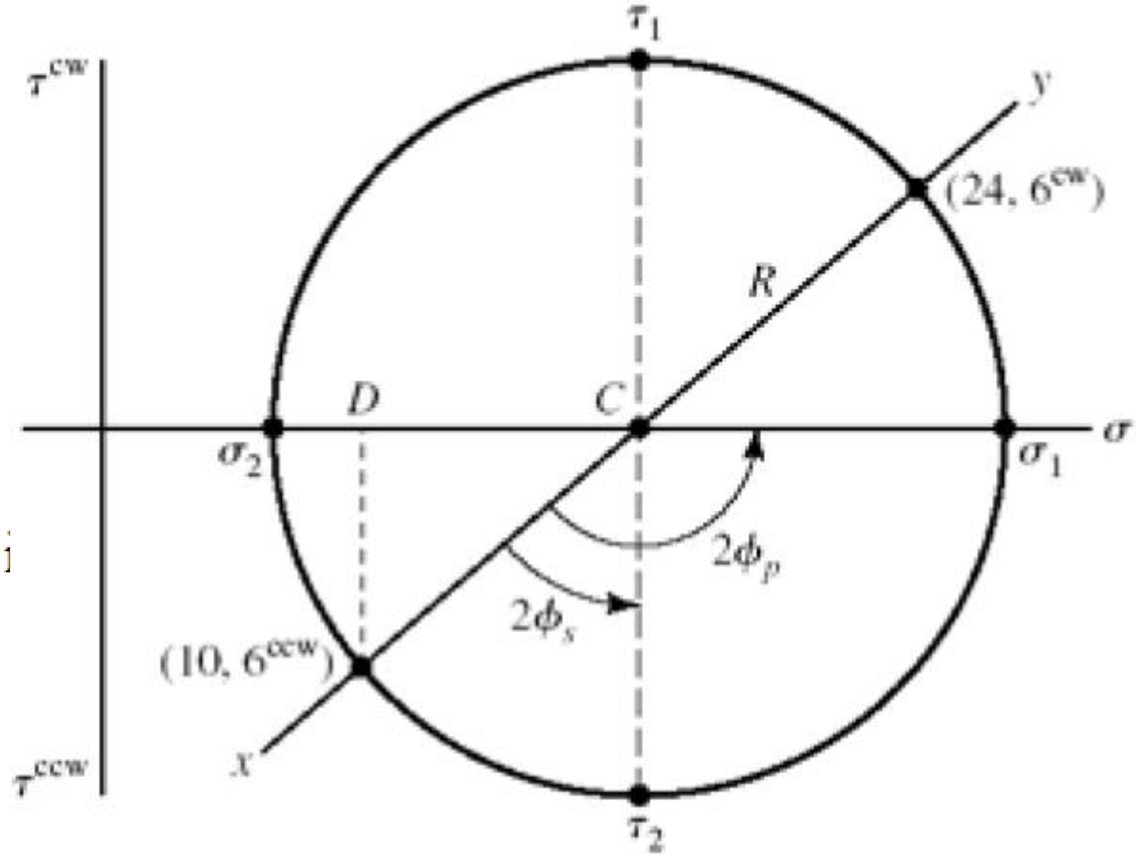
$$C = \frac{24+10}{2} = 17 \text{ kpsi}$$

$$CD = \frac{24-10}{2} = 7 \text{ kpsi}$$

$$R = \sqrt{7^2 + 6^2} = 9.22 \text{ kpsi}$$

$$\sigma_1 = 17 + 9.22 = 26.22 \text{ kpsi}$$

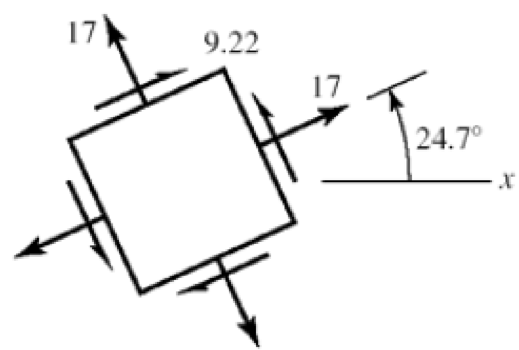
$$\sigma_2 = 17 - 9.22 = 7.78 \text{ kpsi}$$

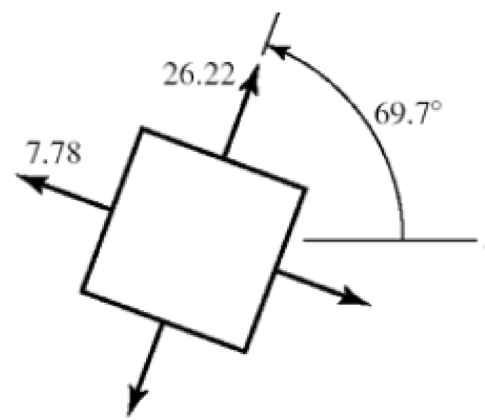


$$\phi_p = \frac{1}{2}\left[90^\circ + \tan^{-1}\left(\frac{7}{6}\right)\right] = 69.7^\circ \text{ ccw}$$

$$\tau_1 = R = 9.22 \text{ kpsi}$$

$$\phi_s = 69.7^\circ - 45^\circ = 24.7^\circ \text{ ccw}$$





(d)

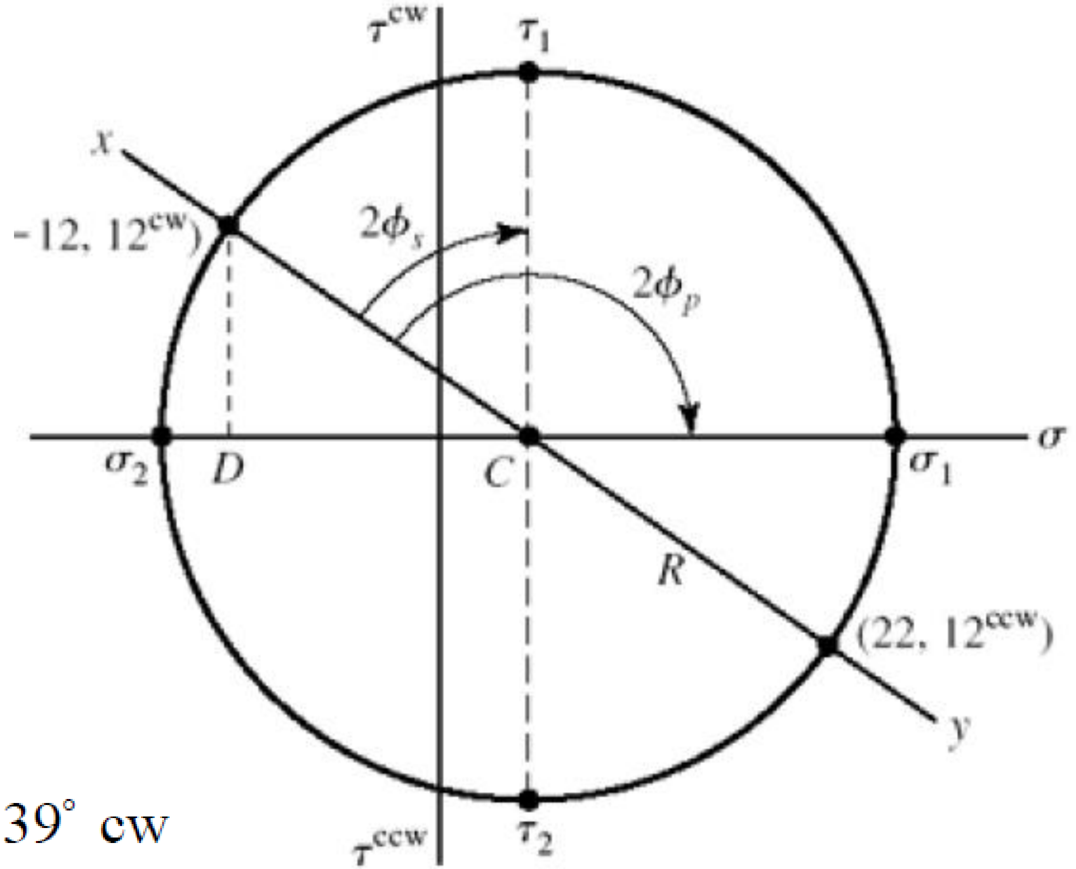
$$C = \frac{-12+22}{2} = 5 \text{ kpsi}$$

$$CD = \frac{12+22}{2} = 17 \text{ kpsi}$$

$$R = \sqrt{17^2 + 12^2} = 20.81 \text{ kpsi}$$

$$\sigma_1 = 5 + 20.81 = 25.81 \text{ kpsi}$$

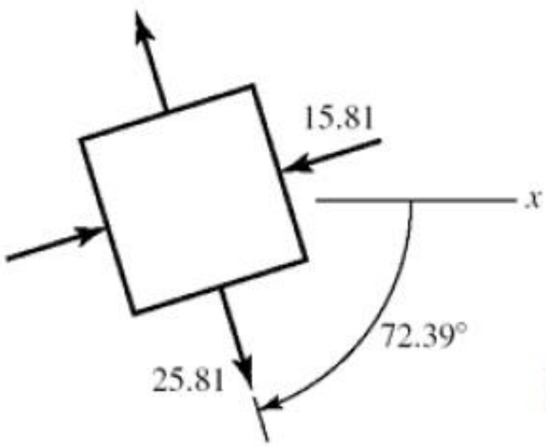
$$\sigma_2 = 5 - 20.81 = -15.81 \text{ kpsi}$$

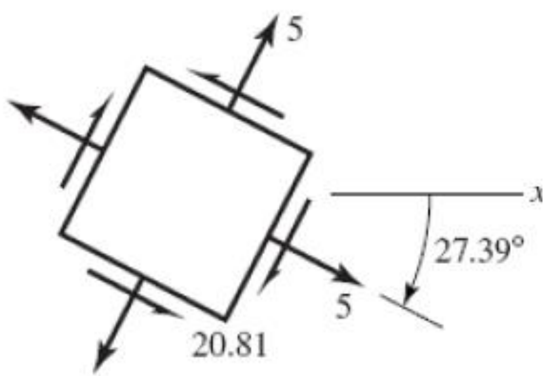


$$\phi_p = \frac{1}{2}\left[90^\circ + \tan^{-1}\left(\frac{17}{12}\right)\right] = 72.39^\circ \text{ cw}$$

$$\tau_1 = R = 20.81 \text{ kpsi}$$

$$\phi_s = 72.39 - 45 = 27.39^\circ$$





Ch

Homework

For each of the plane stress states listed below, draw a Mohr's circle diagram properly labeled, find the principal normal and shear stresses, and determine the angle from the x axis to σ_1. Draw stress elements as in Fig. 3–11c and d and label all details.

(*a*) $\sigma_x = -8\,\text{MPa},\ \sigma_y = 7\,\text{MPa},\ \tau_{xy} = 6\,\text{MPa}\ \text{cw}$

(*b*) $\sigma_x = 9\,\text{MPa},\ \sigma_y = -6\,\text{MPa},\ \tau_{xy} = 3\,\text{MPa}\ \text{cw}$

(*c*) $\sigma_x = -4\,\text{MPa},\ \sigma_y = 12\,\text{MPa},\ \tau_{xy} = 7\,\text{MPa}\ \text{ccw}$

(*d*) $\sigma_x = 6\,\text{MPa},\ \sigma_y = -5\,\text{MPa},\ \tau_{xy} = 8\,\text{MPa}\ \text{ccw}$